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Task Contract N5ori-07806

Nr-026-001

TECHNICAL REPORT No. 62

August 15, 1953

A SUMMARY OF THE NUMERICAL RESULTS OF A THEORETICAL STUDY OF THE SCATTERING OF NEUTRONS BY COMPLEX NUCLEI

By

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and Victor F. Weisskopf

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TABLE OF CONTENTS

SUMMARY OF THE NUMERICAL RESULTS OF A THEORETICAL STUDY OF THE SCATTERING OF NEUTRONS BY COMPLEX NUCLEI

I. INTRODUCTION

A. General Remarks

The purpose of this report is to achieve the presentation in convenient reference form of the numerical results of a theoretical study of the scattering of neutrons by complex nuclei. Only quantities which are plotted or tabulated in this report will be discussed here. Publication in the Physical Review of a paper covering in more detail the physically important results of this study is being planned for the near future. The numerical computations for this report were carried out by Elgie Ginsburgh, Barbara Levine, and Hannah Wasserman of the Joint Computing Group. Grace Rowe performed the necessary drafting work.

B. Total Cross Section

Under the assumption that there is no potential acting on a neutron which is outside of a target nucleus, both the total cross section and the differential cross section for elastic scattering can be expressed in a form such that the entire dependence of these cross sections on target properties is contained in the logarithmic derivative of the neutron wave function evaluated at the surface of the target nucleus. Considering the neutron to have an *effective* spin of zero, one can write the total cross section as*

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* J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952) Chapter VIII.

$$\frac{\sigma_t}{\pi R^2} = \sum_{\ell=0}^{\infty} \frac{\sigma_t^{(\ell)}}{\pi R^2}$$

$$\frac{\sigma_t^{(\ell)}}{\pi R^2} = \frac{2\ell+1}{x^2}\, 2(\mathrm{Re}(1-\eta_\ell)) \,, \tag{1}$$

where x=kR. R is the nuclear radius, and k is the wave number of the incident neutron. η_ℓ is given by the equation

$$\eta_\ell = e^{-2i\delta_\ell}\left(1 - \frac{2s_\ell}{M_\ell + iN_\ell}\right) \quad , \tag{2}$$

with

$$M_\ell = -\mathrm{Im} f_\ell + s_\ell \;,$$
$$N_\ell = \mathrm{Re} f_\ell - \Delta_\ell \;. \tag{3}$$

f_ℓ is the logarithmic derivative of the product of the neutron radial coordinate and the radial wave function evaluated at the surface of the target nucleus. Δ_ℓ , s_ℓ , and δ_ℓ depend upon x only and are given by the expressions

$$\Delta_\ell + i s_\ell = 1 + \frac{x h_\ell'(x)}{h_\ell(x)} \,,$$
$$\delta_\ell = \tan^{-1}\left(-\frac{j_\ell(x)}{n_\ell(x)}\right) \;. \tag{4}$$

Here h_ℓ is a spherical Hankel function, and j_ℓ and n_ℓ are spherical Bessel functions.

Combining Eq. (2) with Eqs. (1) yields

$$\frac{\sigma_t^{(\ell)}}{\pi R^2} = \frac{4}{x^2}(2\ell+1)\left[\sin^2\delta_\ell + s_\ell\,\frac{M_\ell\cos 2\delta_\ell - N_\ell\sin 2\delta_\ell}{M_\ell^2+N_\ell^2}\right],$$

$$\frac{\sigma_t}{\pi R^2} = \sum_{\ell=0}^{\infty}\frac{\sigma_t^{(\ell)}}{\pi R^2}. \qquad (5)$$

Once f_ℓ has been specified via some nuclear model, Eqs. (3) and (5) together with the tables of Section VI of this report constitute a complete base for the numerical computation of the neutron total cross section.

In Sections II and III plots of $\sigma_t/\pi R^2$ are given for a nuclear model specified by the potential function (r is the neutron radial coordinate)

$$V(r) = \begin{cases} -V_0(1+i\zeta), & r < R \\ 0, & r > R \end{cases}. \qquad (6)$$

This gives an f_ℓ of the form

$$f_\ell = 1 + \frac{Xj_\ell'(X)}{j_\ell(X)}, \qquad (7)$$

where

$$X^2 = x^2 + X_0^2(1+i\zeta),$$

$$X_0^2 = \frac{2m}{\hbar^2}V_0R^2, \qquad (8)$$

$$x^2 = \frac{2m}{\hbar^2}ER^2,$$

and m is the mass of the neutron*.

---

* Strictly speaking, the reduced mass of the neutron should be used here and all calculations referred to the center-of-mass system. This correction is not important for heavier nuclei.

If we let

$$f_\ell = w_{1\ell} - i w_{2\ell},$$
$$X = X_1 + iX_2 , \tag{9}$$

it is found that the recurrence relations for the w's are

$$w_{1\ell} = \frac{(X_1^2-X_2^2)(\ell - w_{1,\ell-1}) + 2X_1X_2 w_{2,\ell-1}}{(\ell - w_{1,\ell-1})^2 + w_{2,\ell-1}^2} - \ell ,$$

$$w_{2\ell} = \frac{(X_1^2-X_2^2) w_{2,\ell-1} - 2X_1X_2(\ell - w_{1,\ell-1})}{(\ell - w_{1,\ell-1})^2 + w_{2,\ell-1}^2} ,$$

where

$$X_1^2 - X_2^2 = x^2 + X_0^2 ,$$
$$2X_1X_2 = \zeta X_0^2 . \tag{10}$$

To start the recurrence relations we have

$$w_{1,0} = \mathrm{Re}(X\cot X) = \frac{X_1\cot X_1 \,\mathrm{csch}^2 X_2 + X_2 \coth X_2 \csc^2 X_2}{\cot^2 X_1 + \coth^2 X_2} ,$$

$$w_{2,0} = -\mathrm{Im}(X\cot X) = \frac{X_1 \coth X_2 \csc^2 X_1 - X_2 \cot X_1 \,\mathrm{csch}^2 X_2}{\cot^2 X_1 + \coth^2 X_2} , \tag{11}$$

with X_1 and X_2 given by the equations

$$X_1 = (\tfrac{1}{2}(x^2+X_0^2) + \tfrac{1}{2}((x^2+X_0^2)^2 + (\zeta X_0^2)^2)^{1/2})^{1/2} ,$$
$$X_2 = (-\tfrac{1}{2}(x^2+X_0^2) + ((x^2+X_0^2)^2 + (\zeta X_0^2)^2)^{1/2})^{1/2} . \tag{12}$$

In all the plots of Sections II and III the value of ζ is $\zeta = 0.05$. This value seemed to give a reasonably good representation of the experimental data on total cross sections reported by Barschall*. The values of x range from 0.1 to 3.0,

* H. H. Barschall, Phys. Rev. 86, 431 (1952).

and those of X_0^2 range from 2.5 to 80. In general, the computed values of the total cross section are accurate to better than five per cent, and in most cases the uncertainty is less than three per cent.

C <u>Differential Cross Section for Elastic Scattering</u>

The relevant formula for the differential cross section for elastic scattering is*

$$\frac{1}{\pi R^2}\frac{d\sigma_{el}}{d\Omega} = \frac{1}{x^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)^{1/2}(1-\eta_\ell)Y_{\ell,0}(\theta)\right|^2, \tag{13}$$

which can be written

$$\frac{1}{\pi R^2}\frac{d\sigma_{el}}{d\Omega} = \frac{1}{x^2}\left[(\mathrm{Re}\,\Sigma)^2 + (\mathrm{Im}\,\Sigma)^2\right], \tag{14}$$

where, since $Y_{\ell,0} = (2\ell+1/4\pi)^{1/2}P_\ell(\cos\theta)$,

$$\mathrm{Re}\,\Sigma = (4\pi)^{-1/2}\ 2\sum_{\ell=0}^{\infty}(2\ell+1)\left[\sin^2\delta_\ell + s_\ell\frac{M_\ell\cos 2\delta_\ell - N_\ell\sin 2\delta_\ell}{M_\ell^2+N_\ell^2}\right]P_\ell(\cos\theta),$$

$$= (4\pi)^{-1/2}\frac{x^2}{2}\sum_{\ell=0}^{\infty}\frac{\sigma_t^{(\ell)}}{\pi R^2}P_\ell(\cos\theta), \tag{15}$$

$$\mathrm{Im}\,\Sigma = (4\pi)^{-1/2}\sum_{\ell=0}^{\infty}(2\ell+1)\left[\sin 2\delta_\ell - 2s_\ell\frac{M_\ell\sin 2\delta_\ell + N_\ell\cos 2\delta_\ell}{M_\ell^2+N_\ell^2}\right]P_\ell(\cos\theta).$$

When f_ℓ is given, Eqs. (3), (14), and (15) and the tables of Section VI are sufficient for the computation of the differential cross section for elastic scattering.

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* Blatt and Weisskopf, loc. cit.

In Section IV differential cross sections for elastic scattering are plotted using the f_ℓ of Eq. (7). The ratio of x^2 to X_0^2 is fixed at 1/19 corresponding to 1 Mev incident neutrons with a 19 Mev well depth. ζ is set equal to 0.05, and X_0^2 ranges from 4.75 to 83.79. The uncertainty in computed values is less than five per cent.

The differential cross sections of Section VI do not include the possibility of elastic scattering via compound nucleus formation. By providing a mechanism for compound nucleus formation with elastic reemission analogous to that of Feshbach, Peaslee, and Weisskopf* and including also the possibility of independent particle scattering, it is found that on averaging over the narrow fine-structure resonances the resulting differential cross section for elastic scattering is the same as that of Eq. (14) with the exception of a term of the form

$$\sum_{\ell=0}^{\infty} \frac{\sigma_{ce}^{(\ell)}}{\pi R^2} \left| Y_{\ell,0}(\theta) \right|^2 , \tag{16}$$

which must be added on the right-hand side of Eq. (14) where

$$\frac{\sigma_{ce}^{(\ell)}}{\pi R^2} = \frac{4}{x^2} (2\ell+1) s_\ell \frac{(-\mathrm{Im} f_\ell)}{M_\ell^2+N_\ell^2} , \tag{17}$$

is the partial cross section for compound-elastic scattering⁄. Eq. (17) has the <u>form</u> of a reaction cross section with f_ℓ given by Eq. (9) since no actual absorption is present. The quantity ζ used in the plots of Section V is <u>not</u> an absorption

* H. Feshbach, D. C. Peaslee, and V. F. Weisskopf, Phys. Rev. <u>71</u>, 145 (1947).

⁄ This has been called capture-elastic scattering by some authors.

parameter; it specifies the strength of the coupling between the incident neutron and target nucleus which leads to elastic scattering via the intermediary of compound nucleus formation. No net loss of particles is involved. It is evident from the $|Y_{\ell,0}|^2$ in Eq. (16) that this additional term is symmetric about an angle of 90°.

The two sets of differential cross sections for elastic scattering represent extremes. One set corresponds to absorption and no compound-elastic scattering, while the other corresponds to compound-elastic scattering and no absorption. Both sets, of course, contain the independent-particle features.

Section V contains the plots of the averaged differential cross section. The values of ζ , x^2, and X_0^2 are the same as those used in Section IV. The uncertainty in the computed values is less than five per cent.

D. Tables

Section VI of this report contains tables of quantities important for neutron-cross-section calculations that depend upon x only (see Eq. (4)), i.e. that do not depend upon a specific model for the internal structure of the target nucleus.

One can obtain $\cos 2\delta_\ell$, $\sin 2\delta_\ell$, and $\sin^2\delta_\ell$ either by using tables of δ_ℓ * or by using tables of $j_\ell(x)$ and $n_\ell(x)$ and the formulas

---

* A. N. Lowan, P. M. Morse, H. Feshbach, and M. Lax, Scattering and Radiation from Circular Cylinders and Spheres, February, 1945.

$$\cos 2\delta_\ell = \frac{1-j_\ell^2(x)/n_\ell^2(x)}{1+j_\ell^2(x)/n_\ell^2(x)} \quad ,$$

$$\sin 2\delta_\ell = -\frac{2j_\ell(x)/n_\ell(x)}{1+j_\ell^2(x)/n_\ell^2(x)} \quad , \tag{18}$$

$$\sin^2\delta_\ell = \frac{j_\ell^2(x)/n_\ell^2(x)}{1+j_\ell^2(x)/n_\ell^2(x)} \quad .$$

From Eq. (4) and the recurrence relations for the spherical Hankel functions, it is possible to show that

$$\Delta_\ell = \frac{x^2(\ell-\Delta_{\ell-1})}{(\ell-\Delta_{\ell-1})^2+s_{\ell-1}^2} - \ell \ ,$$

$$s_\ell = \frac{x^2 s_{\ell-1}}{(\ell-\Delta_{\ell-1})^2+s_{\ell-1}^2} \quad . \tag{19}$$

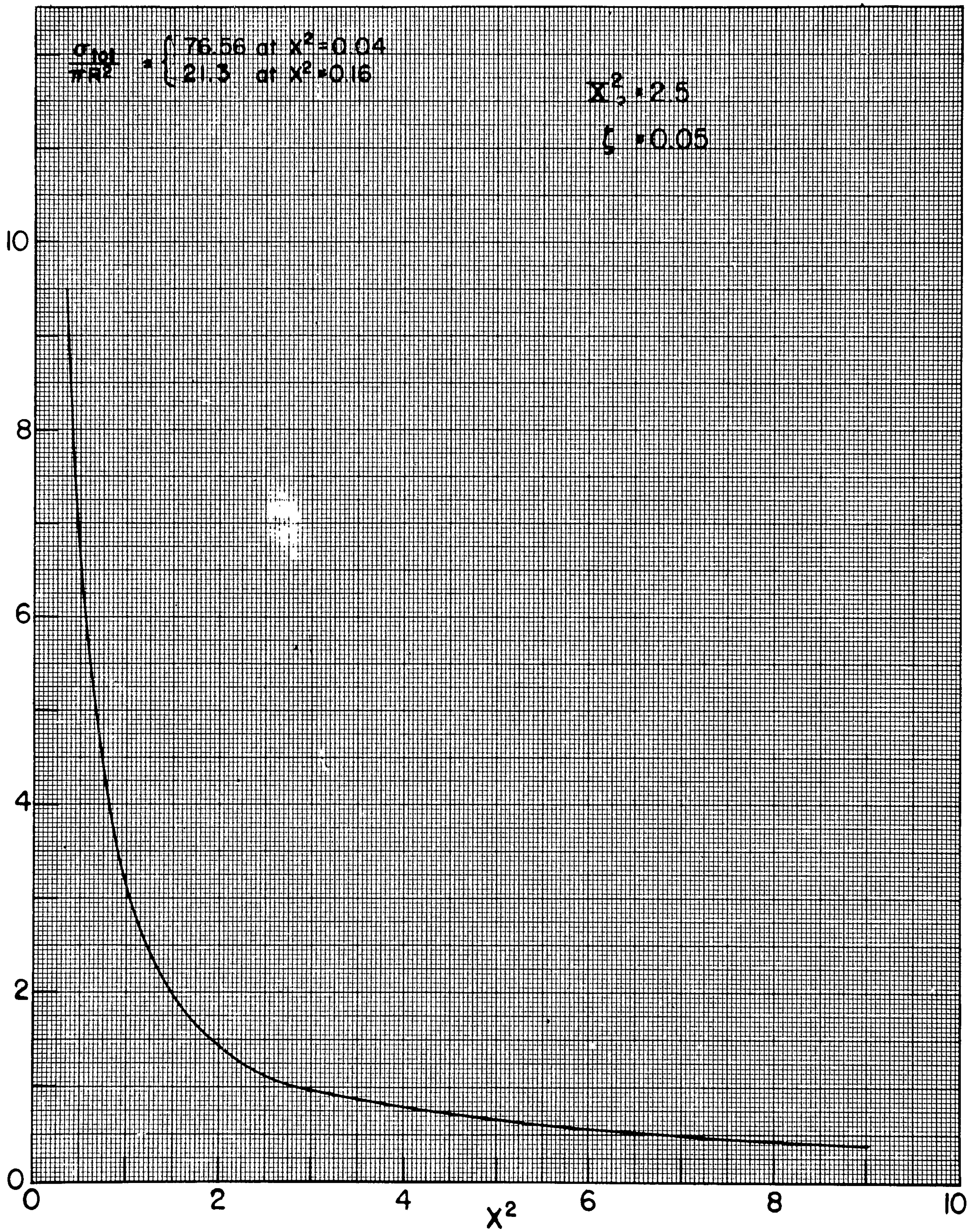
To start these recurrence relations, we have

$$\Delta_o = 0 \ ,$$

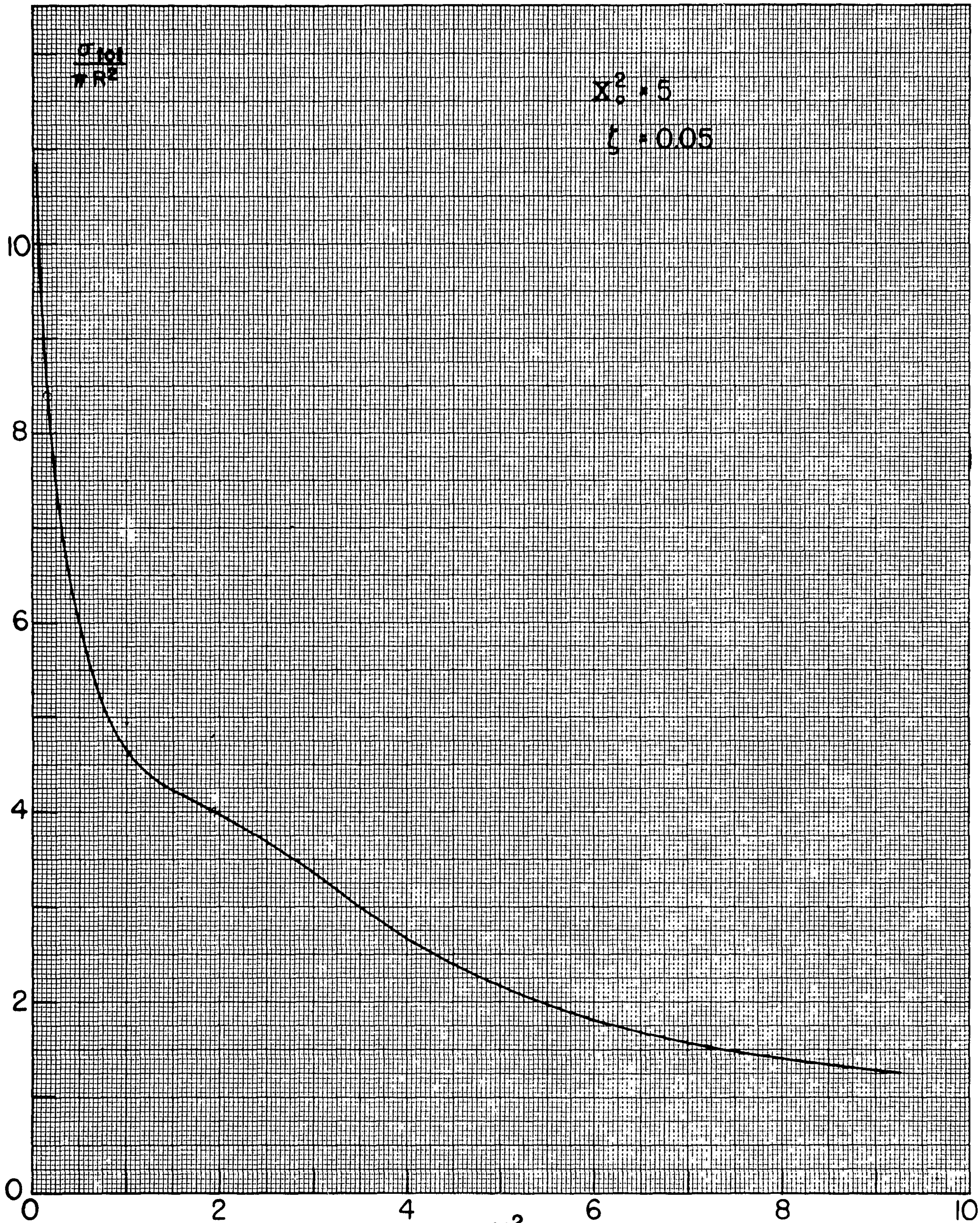
$$s_o = x \ . \tag{20}$$

In Section VI, tables of s_ℓ , Δ_ℓ , $\cos 2\delta_\ell$, $\sin 2\delta_\ell$, and $\sin^2\delta_\ell$ are given for values of x from 0.1 to 3.0 in steps of 0.1 and for values of ℓ from 0 to 7 inclusive. The tables are accurate to ± 1 in the last digit.

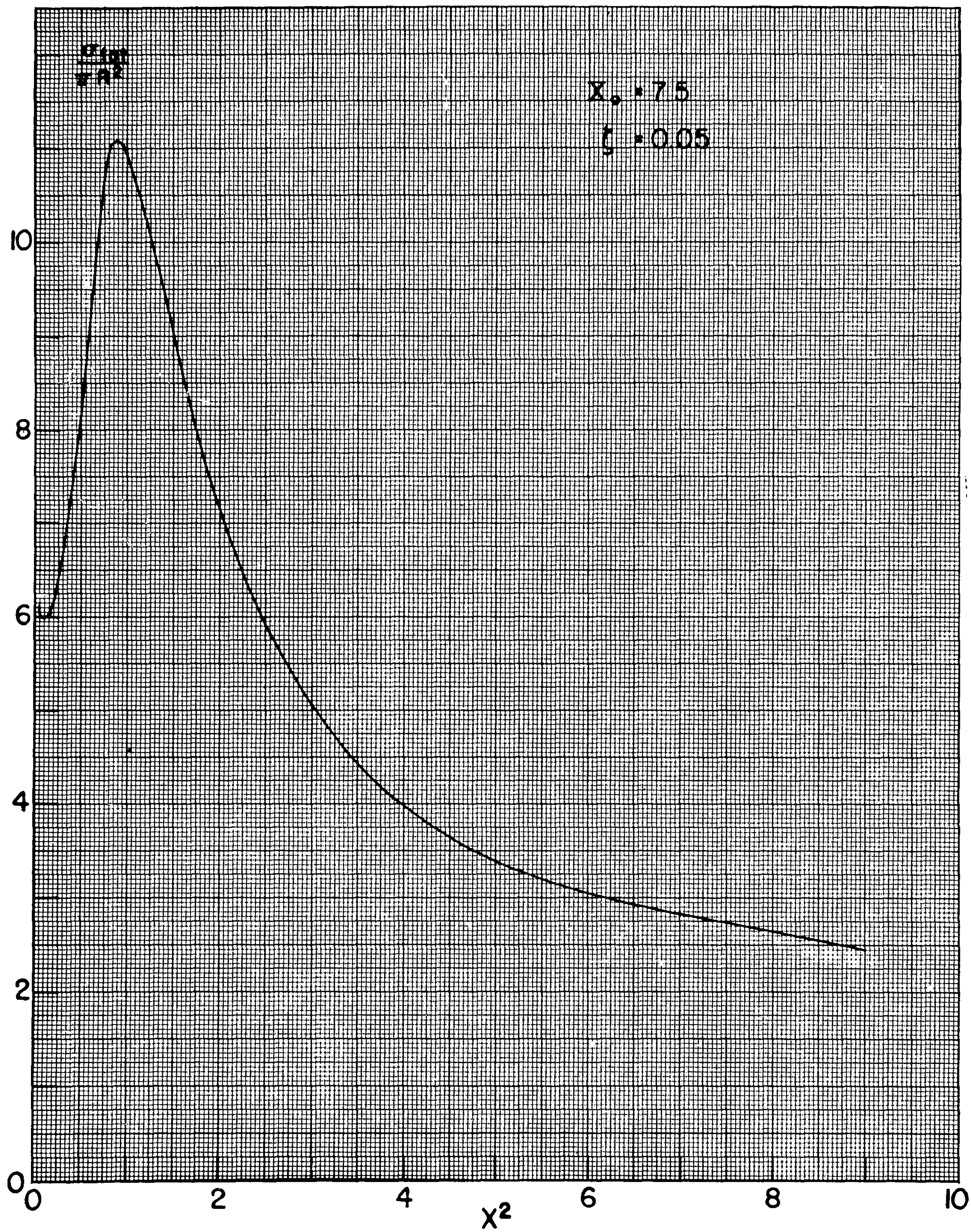
II. PLOTS OF TOTAL CROSS SECTION VS. x^2 FOR FIXED X_o^2 WITH $\zeta = 0.05$

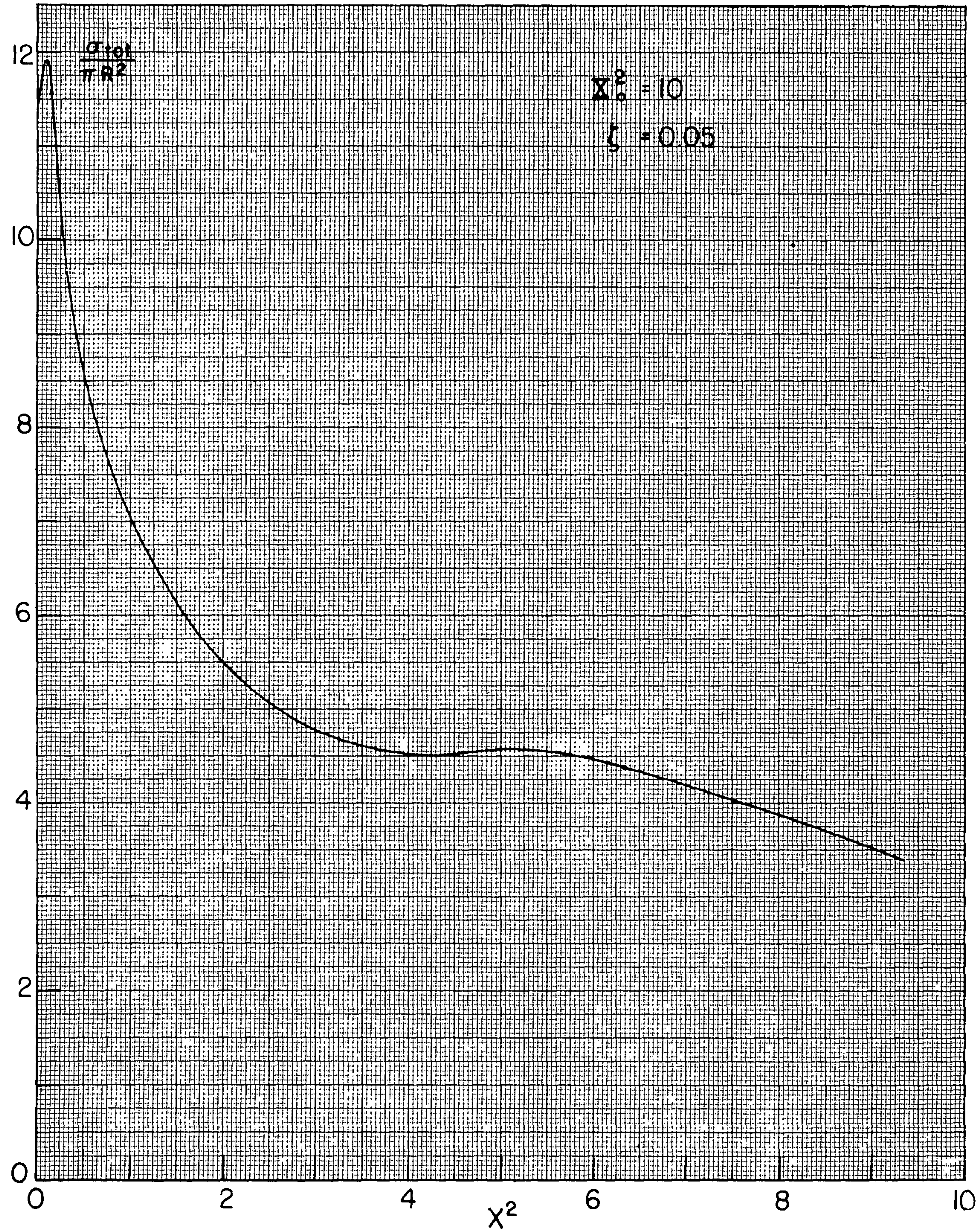


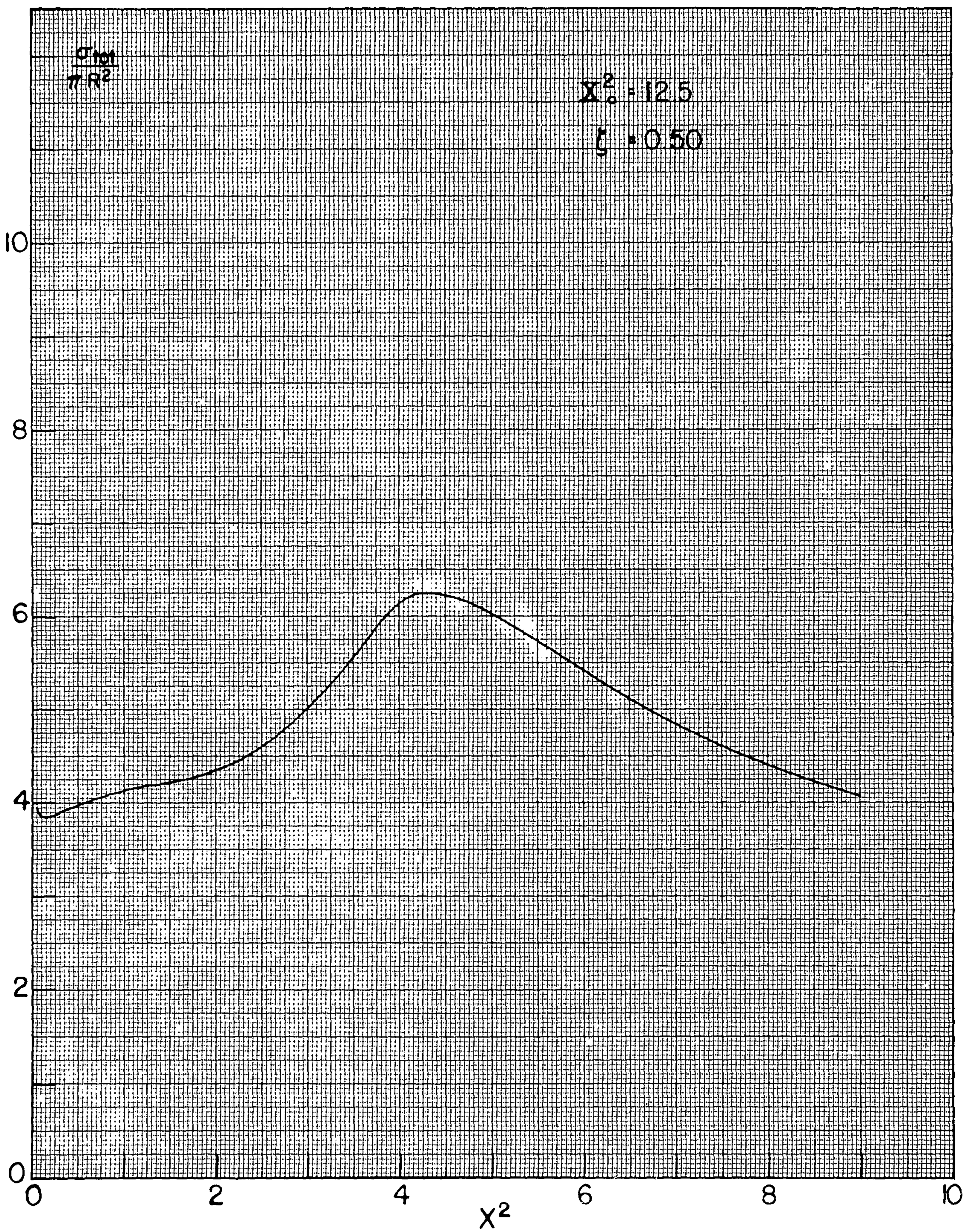
$\frac{\sigma_{tot}}{\pi R^2} = \begin{cases} 76.56 \text{ at } X^2 = 0.04 \\ 21.3 \text{ at } X^2 = 0.16 \end{cases}$
$X_0^2 = 2.5$
$\zeta = 0.05$
0
2
4
6
8
10
X^2



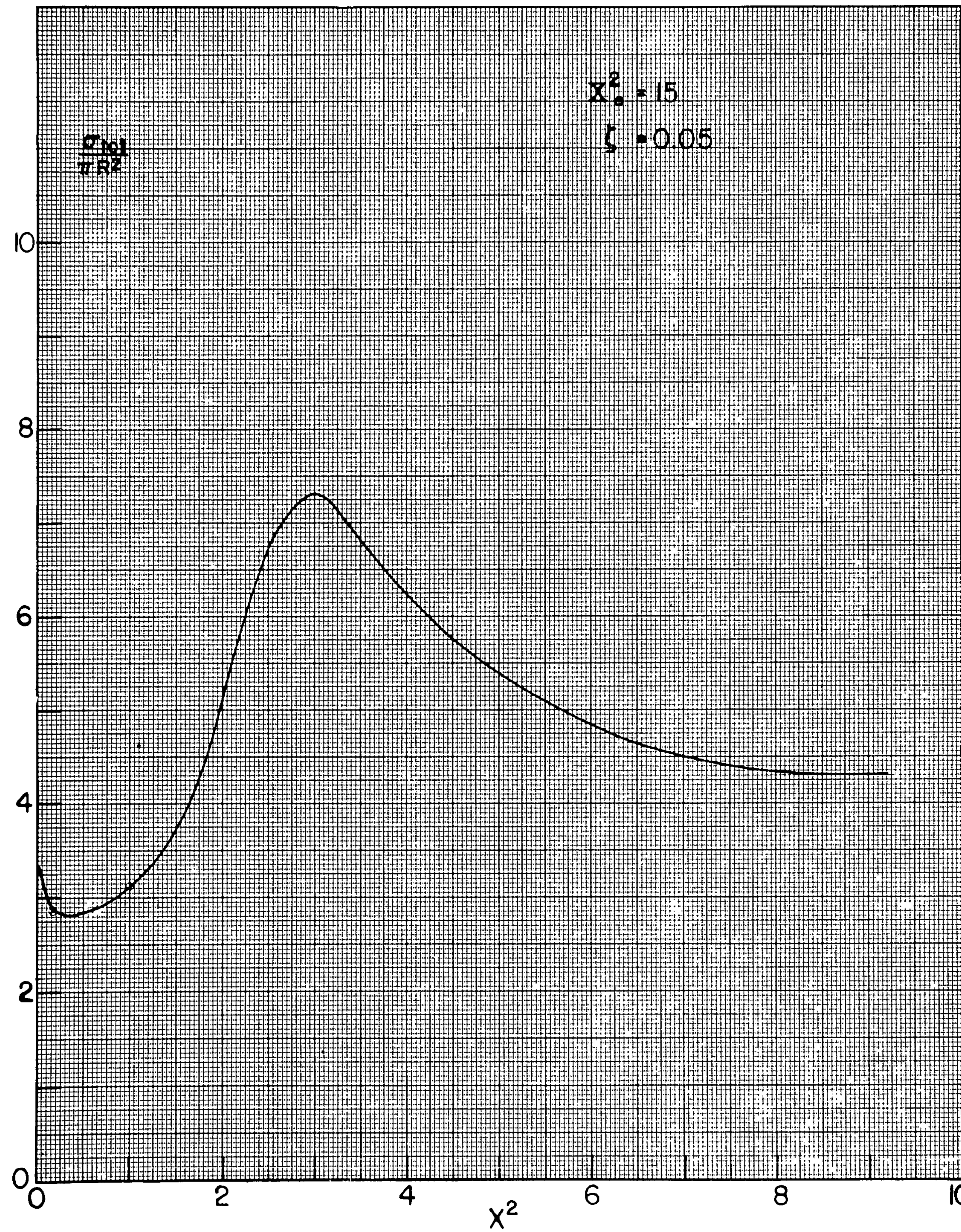
$\frac{\sigma_{101}}{\pi R^2}$
$X_0^2 = 5$
$\xi = 0.05$
0
2
4
6
8
10
X^2

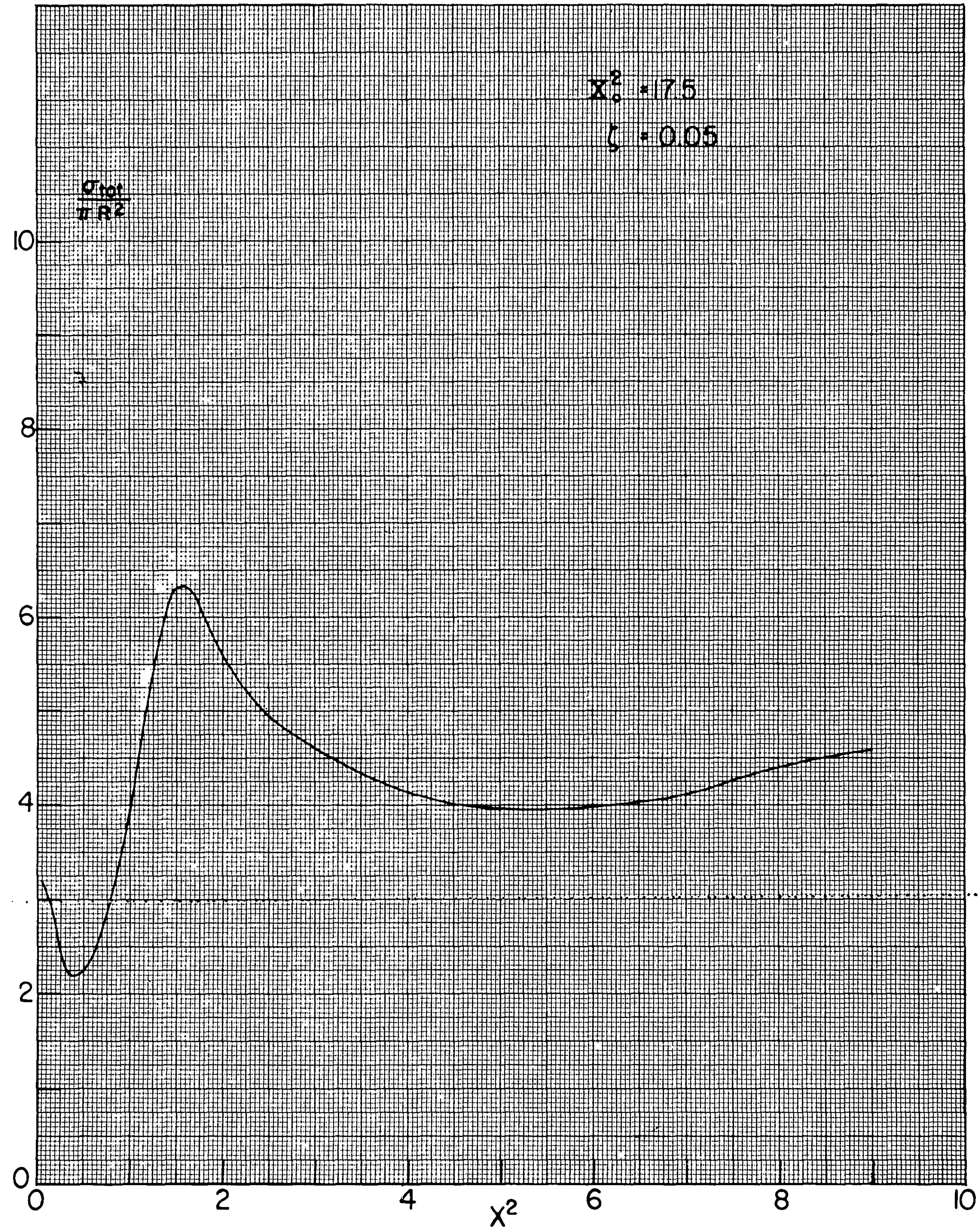

X₀ = 7.5
ζ = 0.05
0
2
4
6
8
10
X²

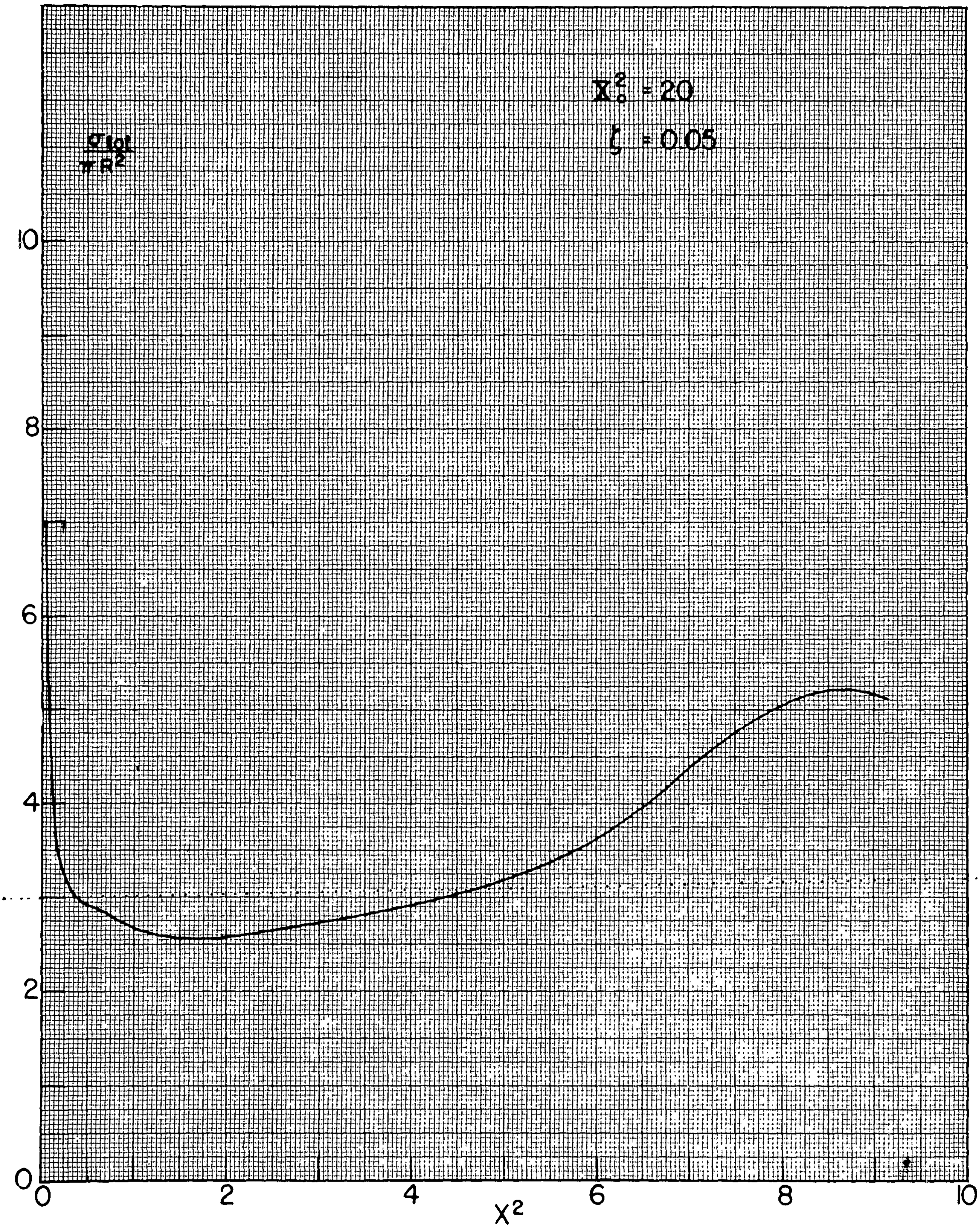

$\frac{\sigma_{tot}}{\pi R^2}$
$X_0^2 = 10$
$\zeta = 0.05$
0
2
4
6
8
10
12
X^2

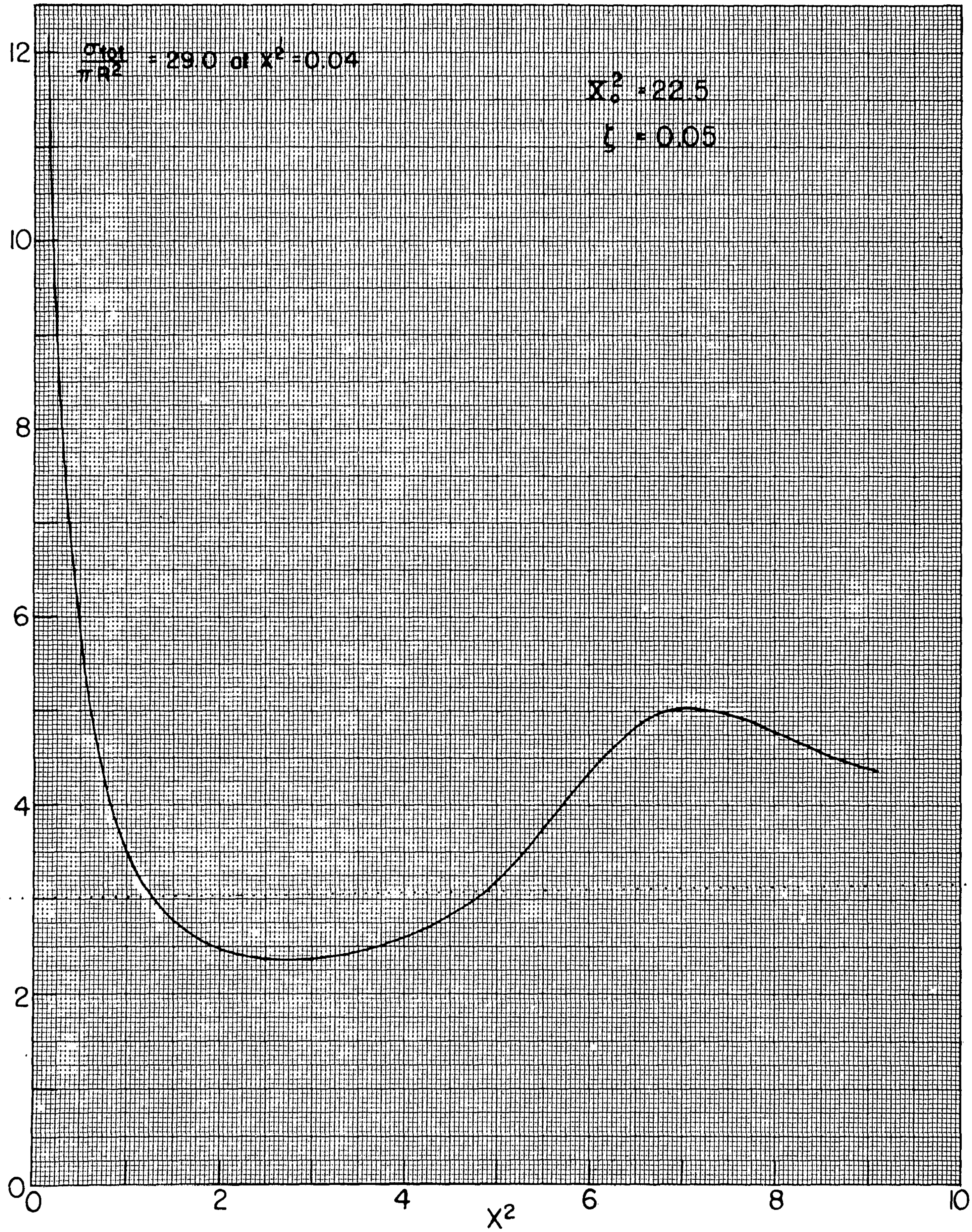


$\frac{\sigma_{tot}}{\pi R^2}$
$X_o^2 = 12.5$
$\zeta = 0.50$
10
8
6
4
2
0
0
2
4
6
8
10
X^2

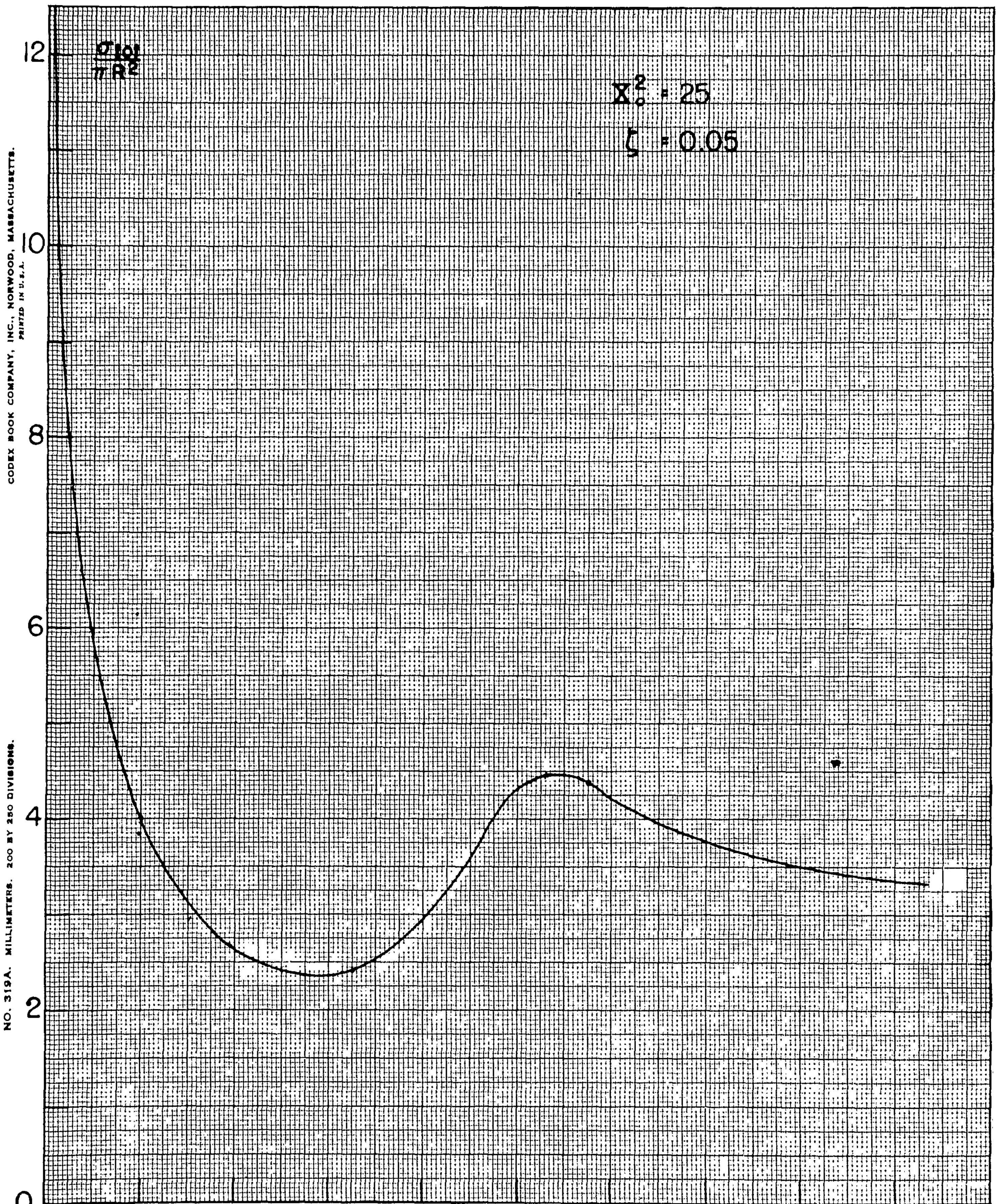

$X_o^2 = 15$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
10
X^2


$X_o^2 = 17.5$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
10
X^2


$X_0^2 = 20$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
10
8
6
4
2
0
0
2
4
6
8
10
X^2



$\frac{\sigma_{tot}}{\pi R^2} = 29.0$ at $X^2 = 0.04$
$X_0^2 = 22.5$
$\zeta = 0.05$
0
2
4
6
8
10
12
X^2



$\frac{\sigma_{101}}{\pi R^2}$
$X_o^2 = 25$
$\zeta = 0.05$
12
10
8
6
4
2
0
0
2
4
6
8
10
X^2
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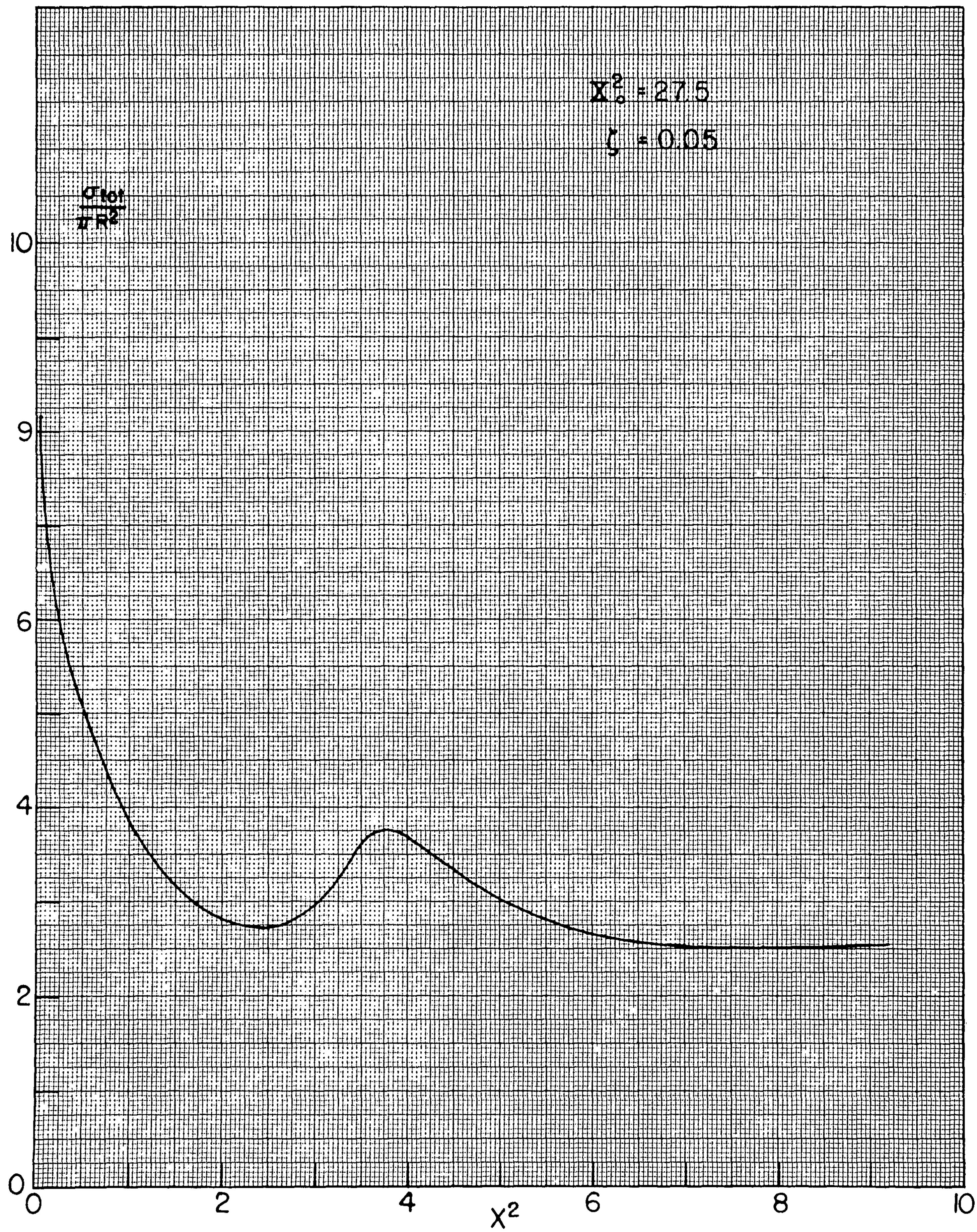
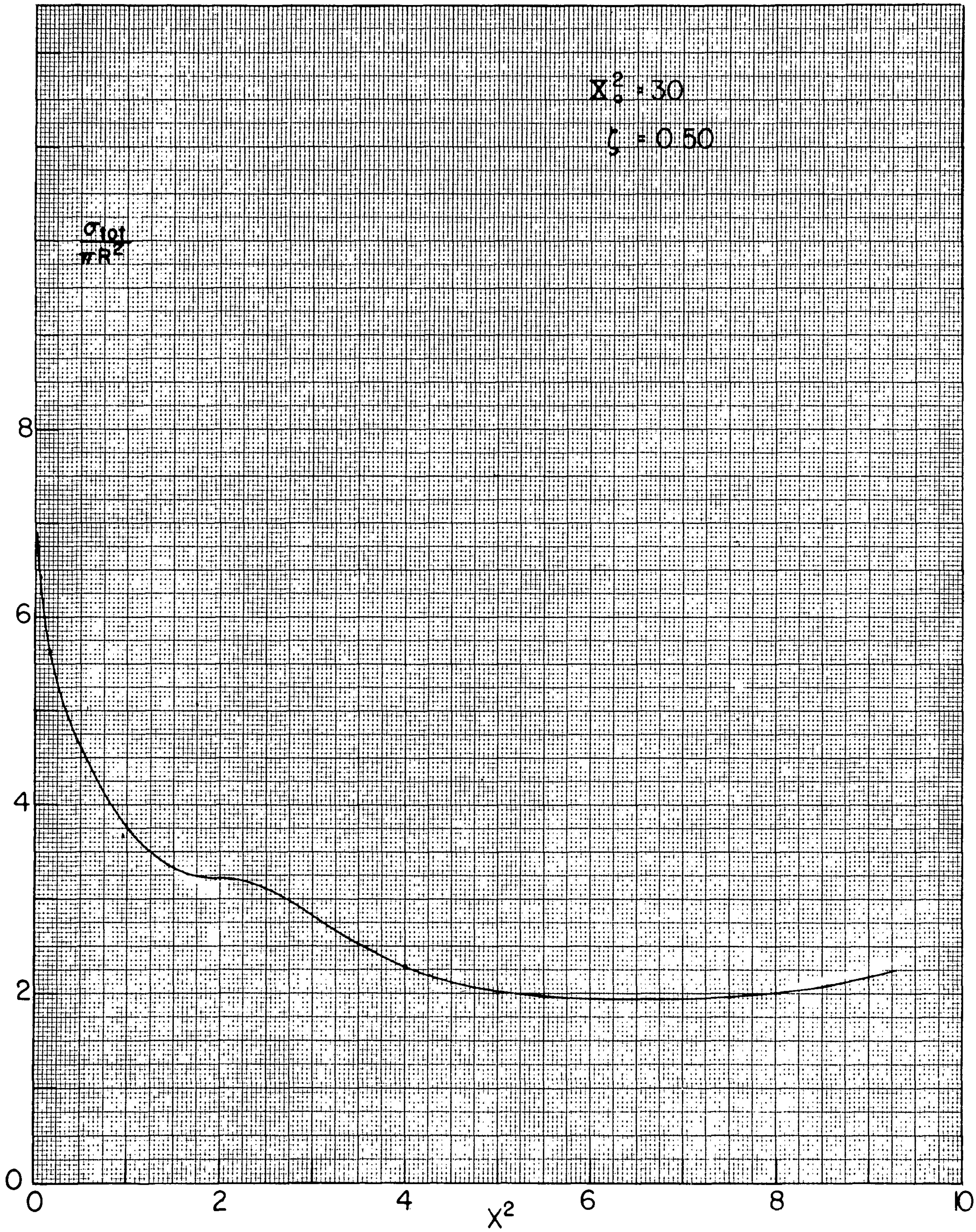
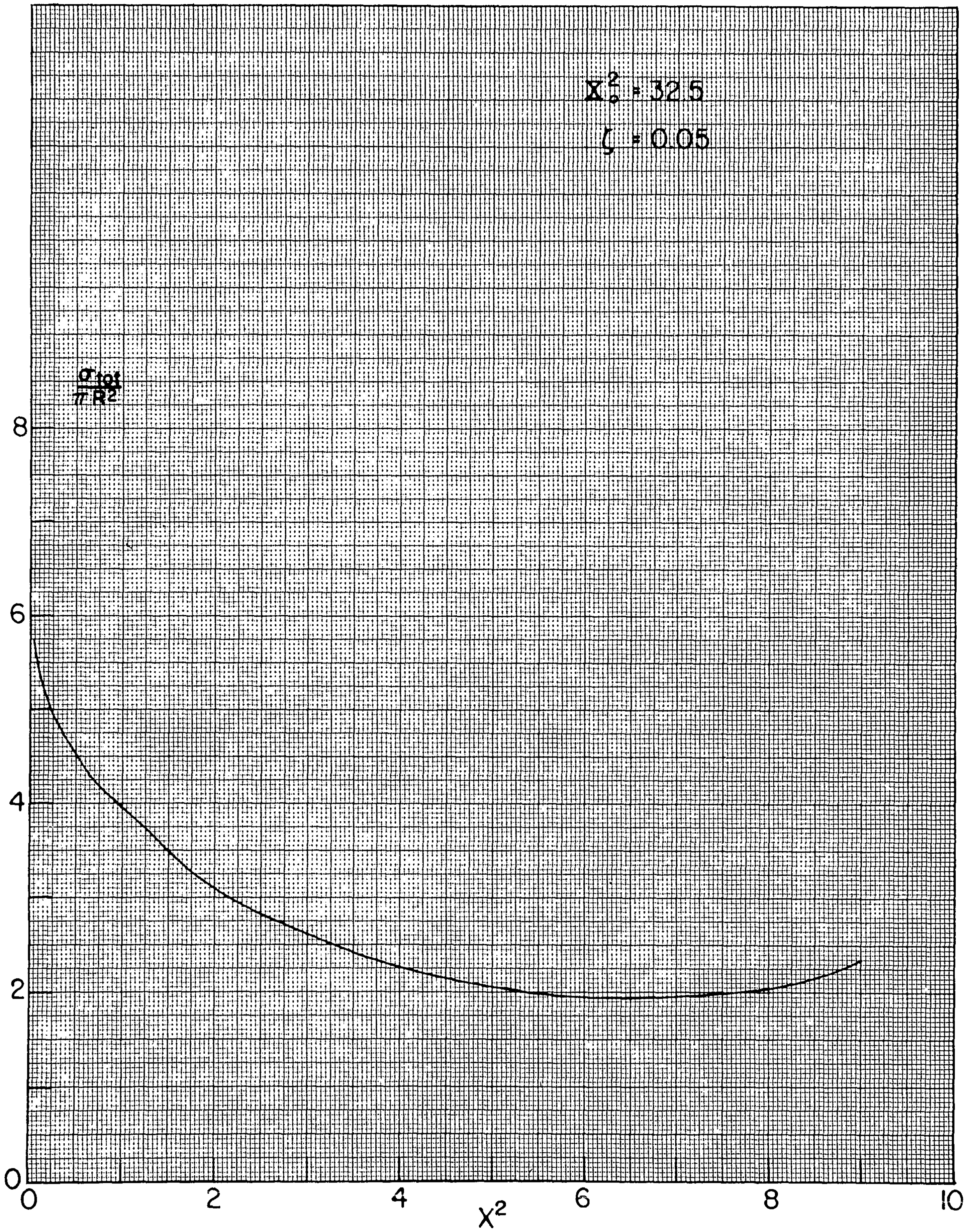
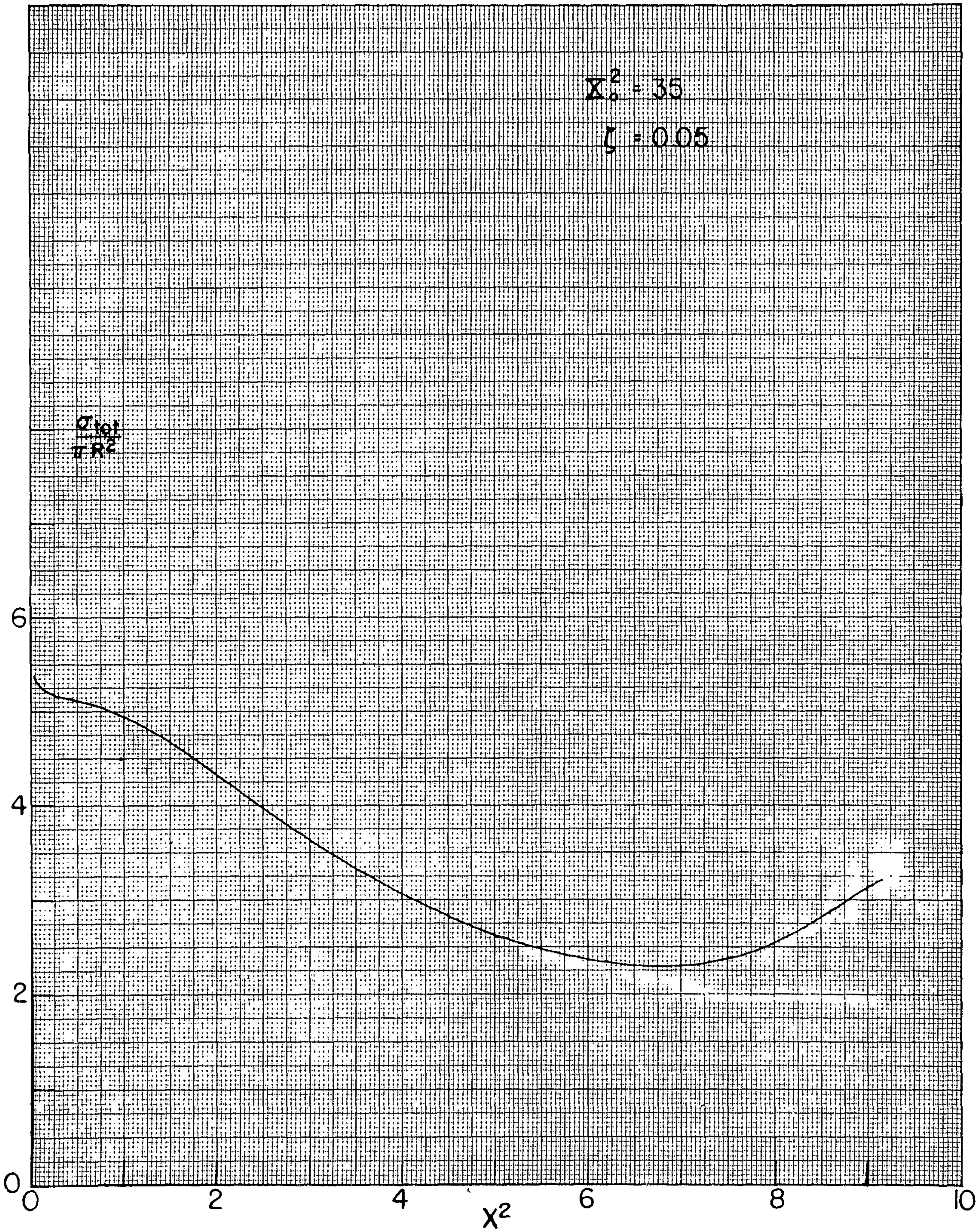

$X_0^2 = 27.5$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
10
9
6
4
2
0
0
2
4
6
8
10
X^2



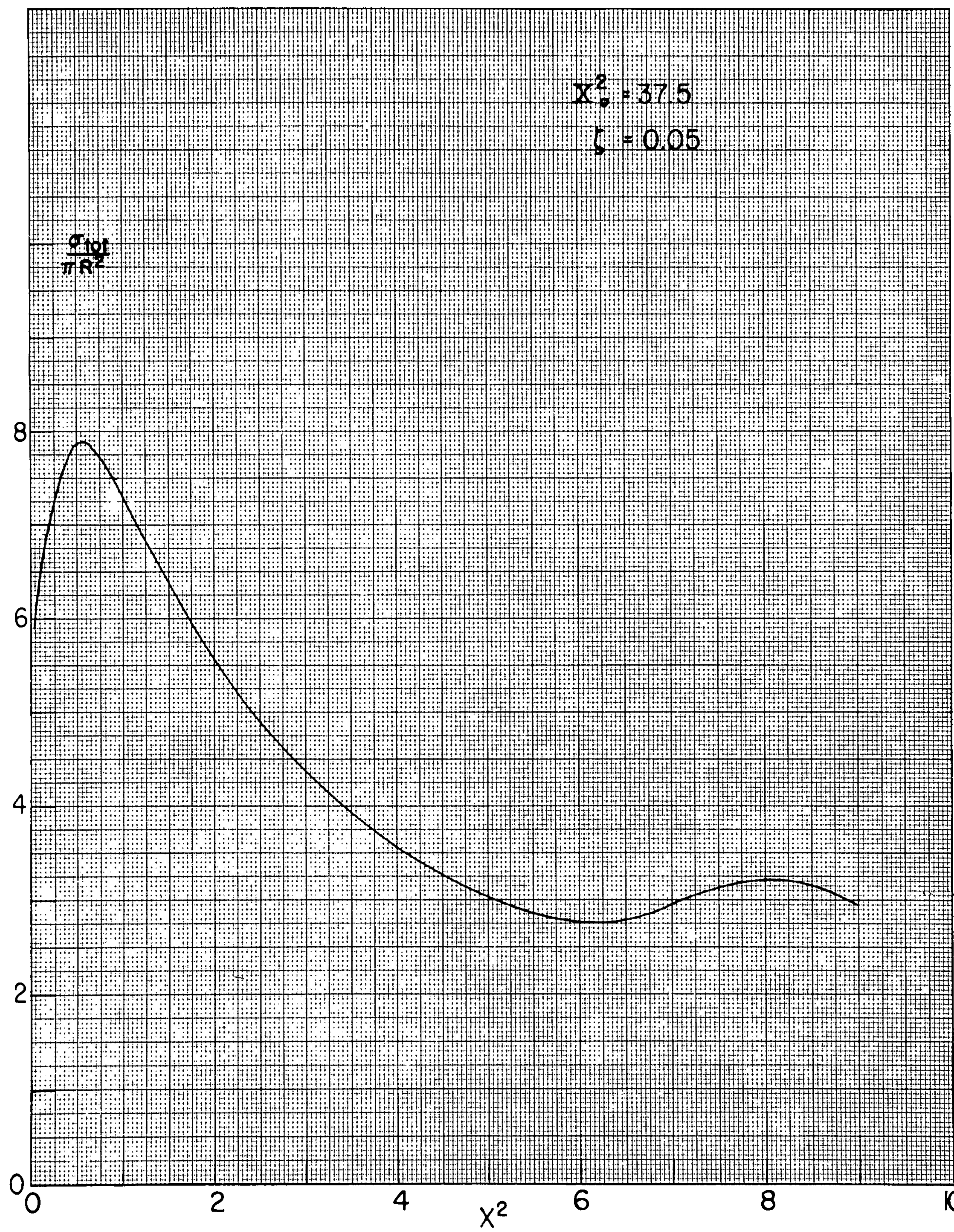
$X_0^2 = 30$
$\zeta = 0.50$
$\frac{\sigma_{tot}}{\pi R^2}$
8
6
4
2
0
0
2
4
6
8
10
X^2



$X_o^2 = 32.5$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
0
2
4
6
8
10
X^2



$X_o^2 = 35$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
10
X^2


$X_0^2 = 37.5$
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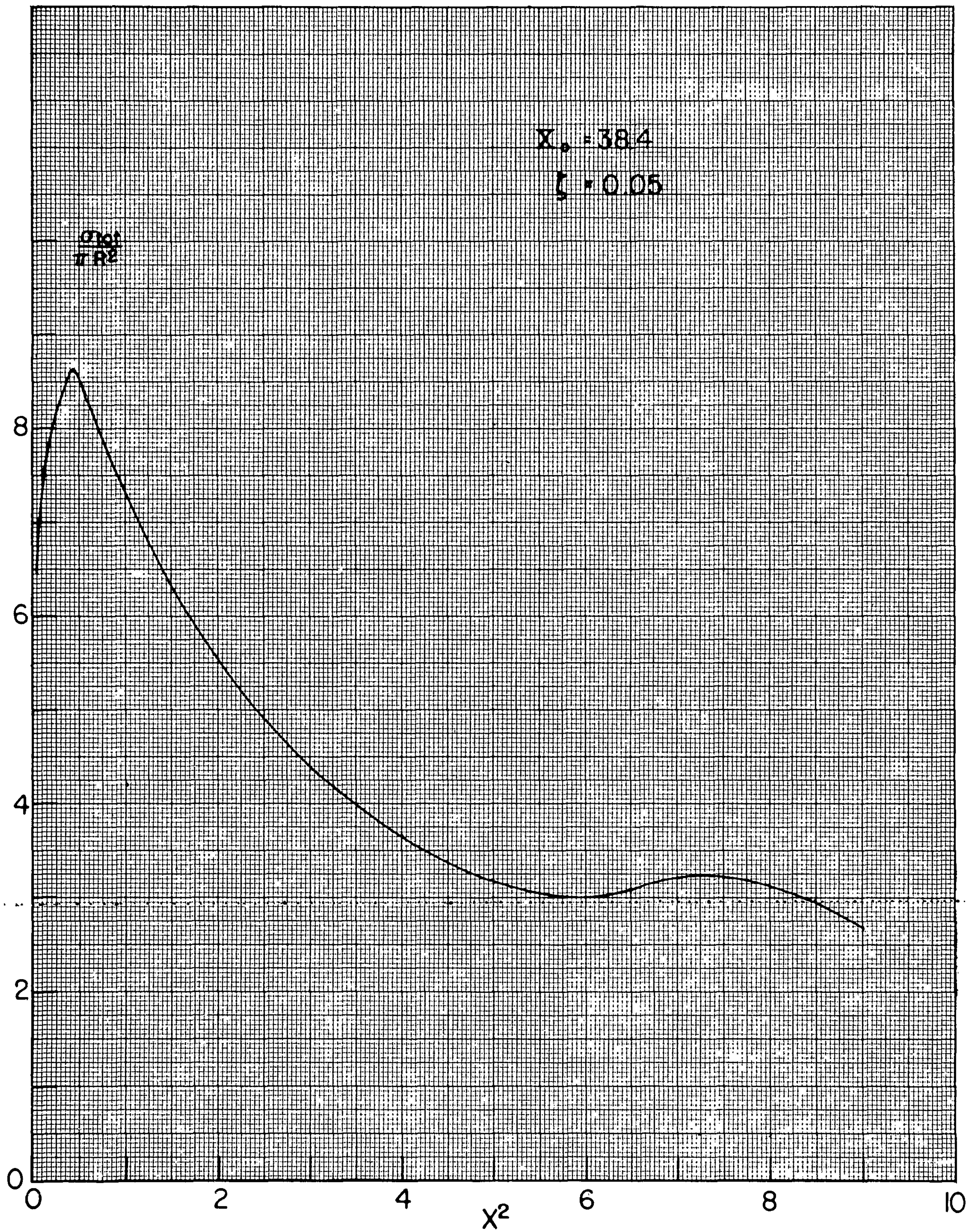
0
2
4
6
8
0
2
4
6
8
10
X^2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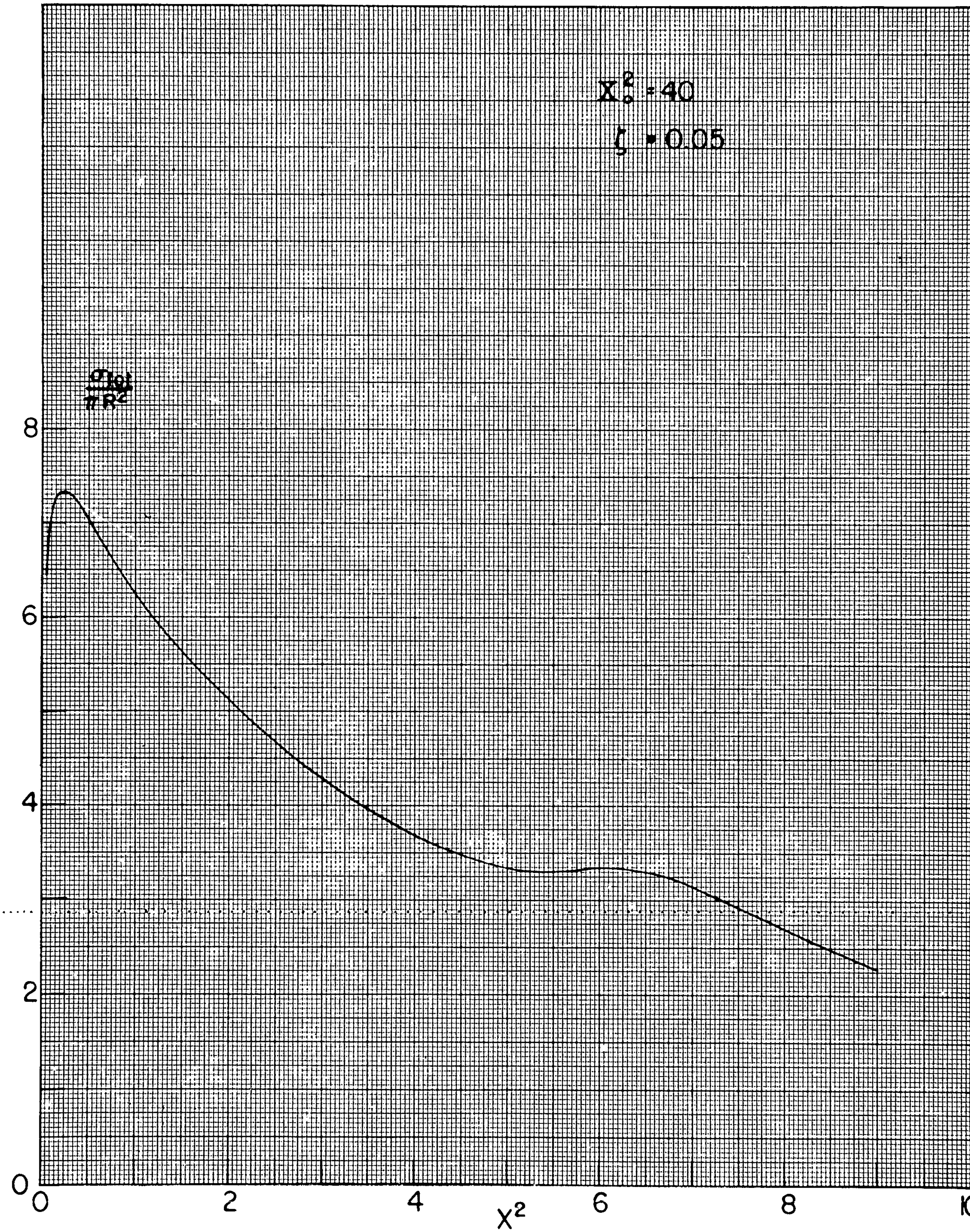
6
8
10
X^2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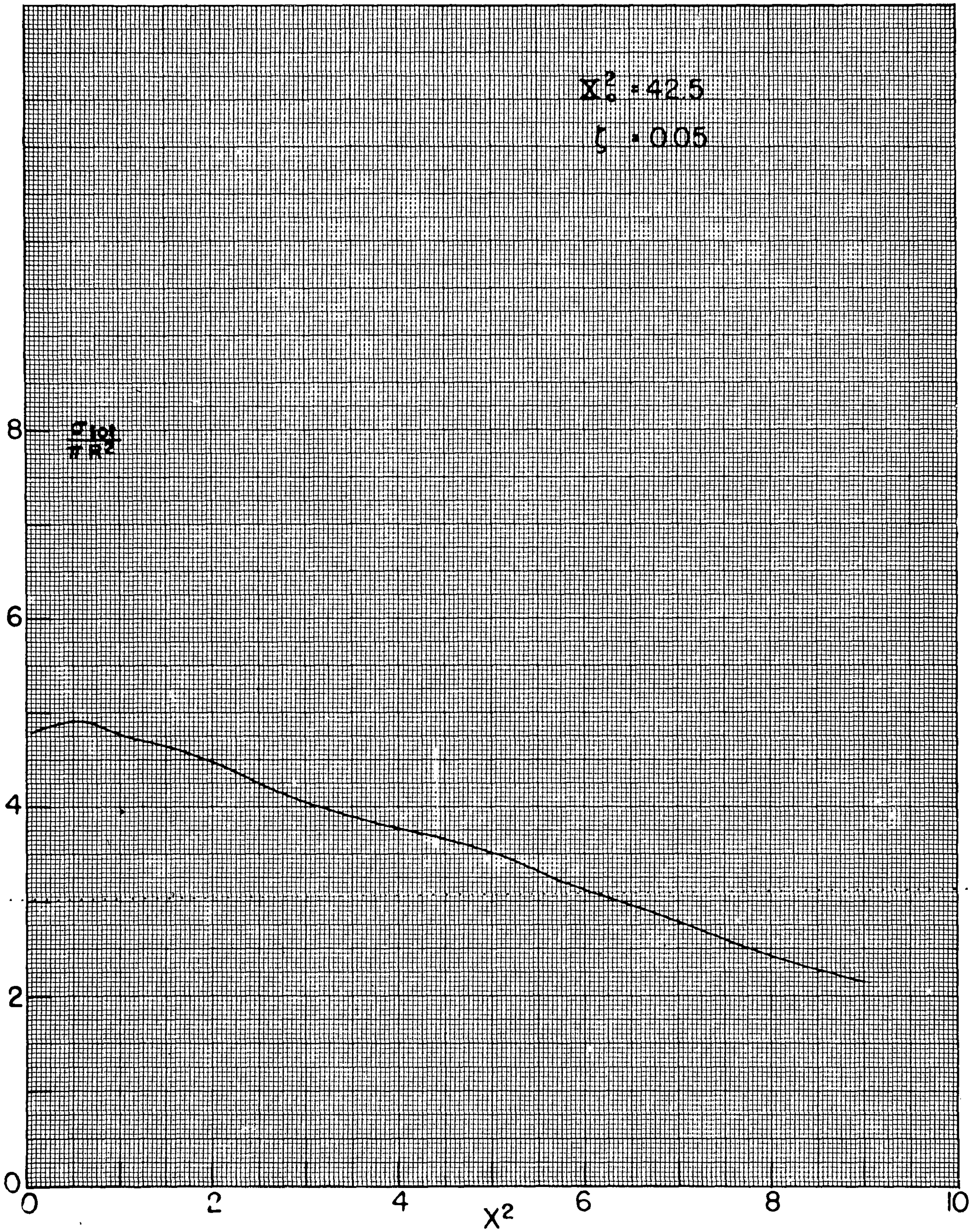
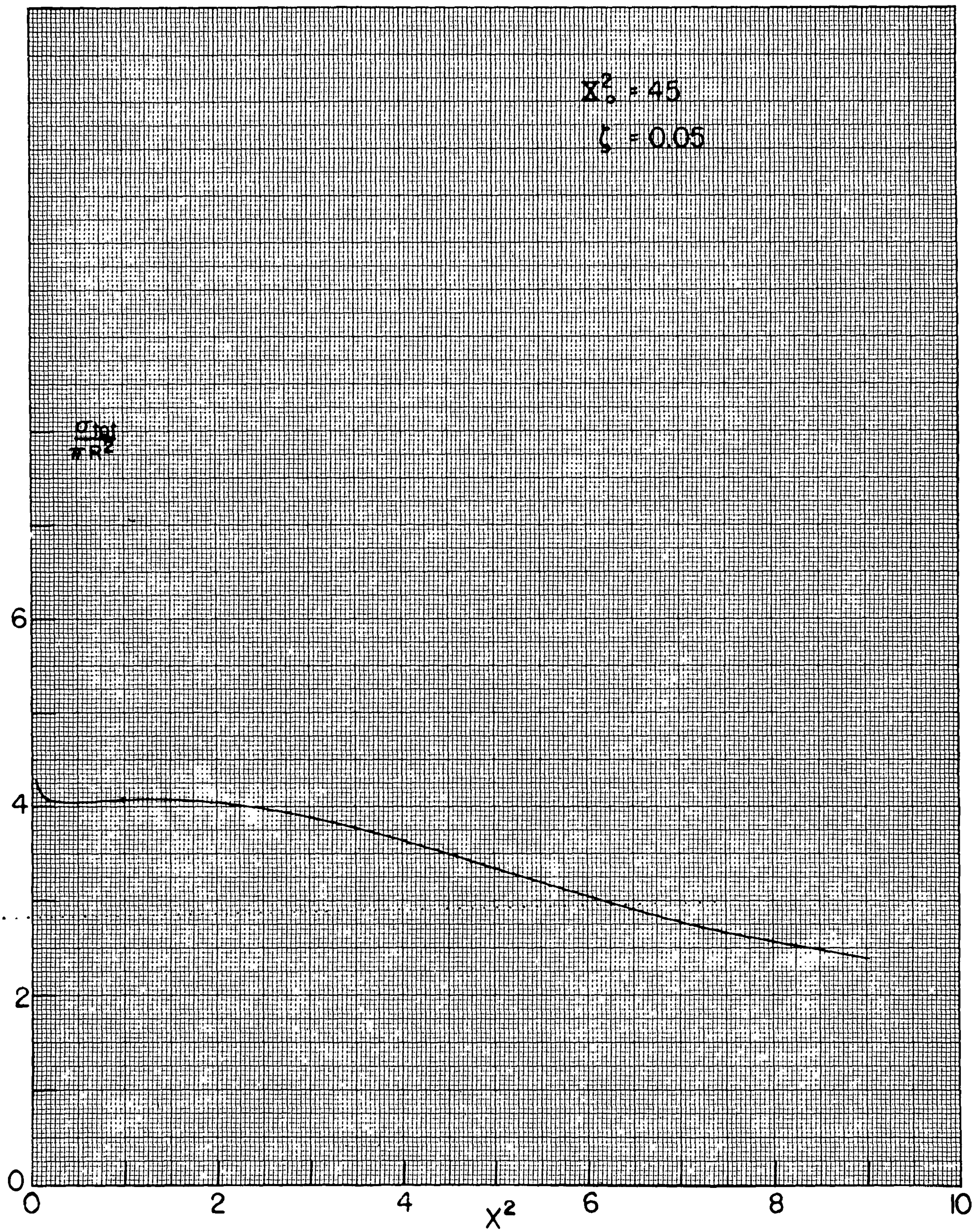
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
0
2
4
6
8
10
X^2



$X_0^2 = 42.5$
$\zeta = 0.05$
$\frac{Q}{\pi}\frac{|\sigma|}{R^2}$
0
2
4
6
8
10
X^2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$\zeta = 0.05$
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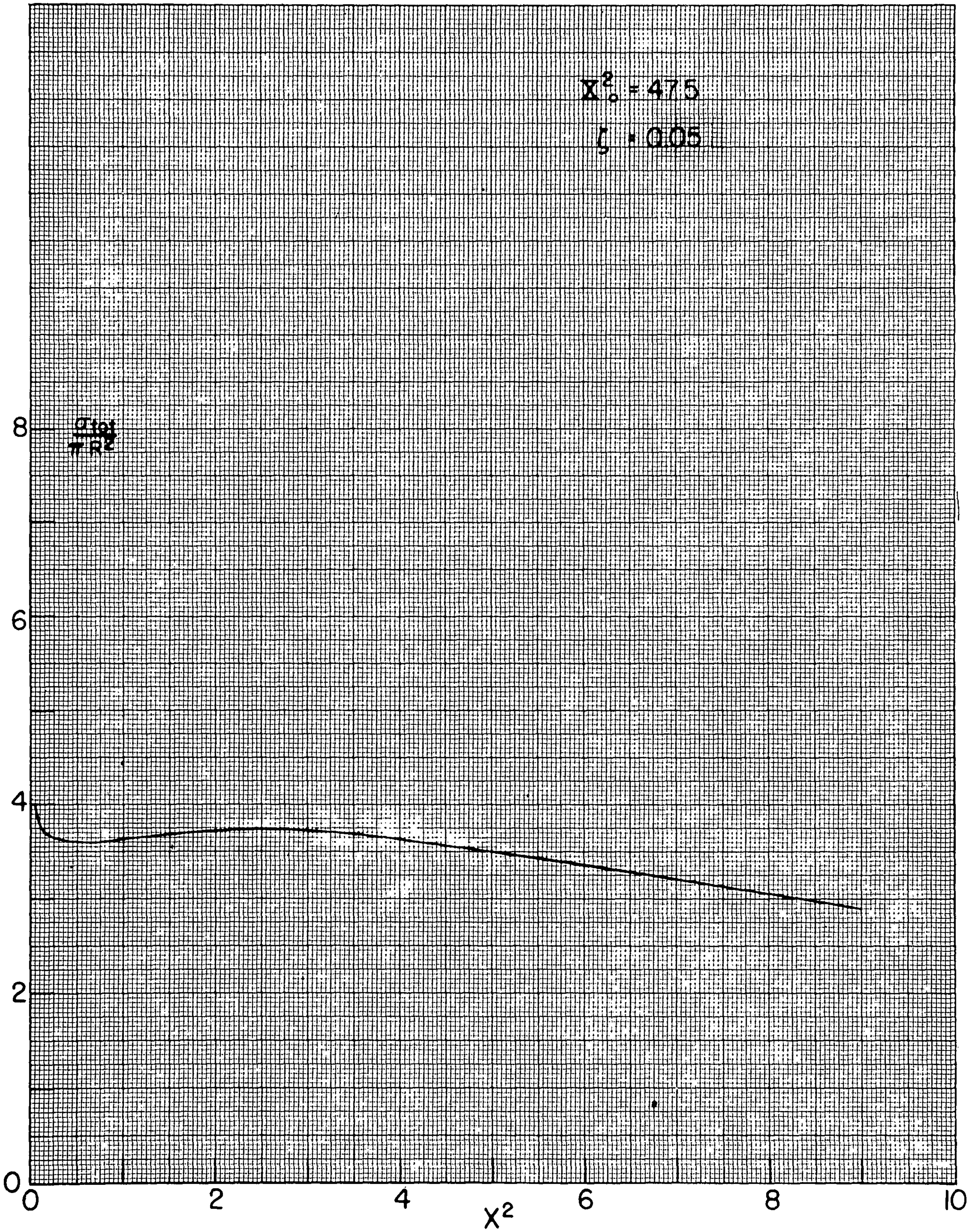
0
2
4
6
0
2
4
6
8
10
X^2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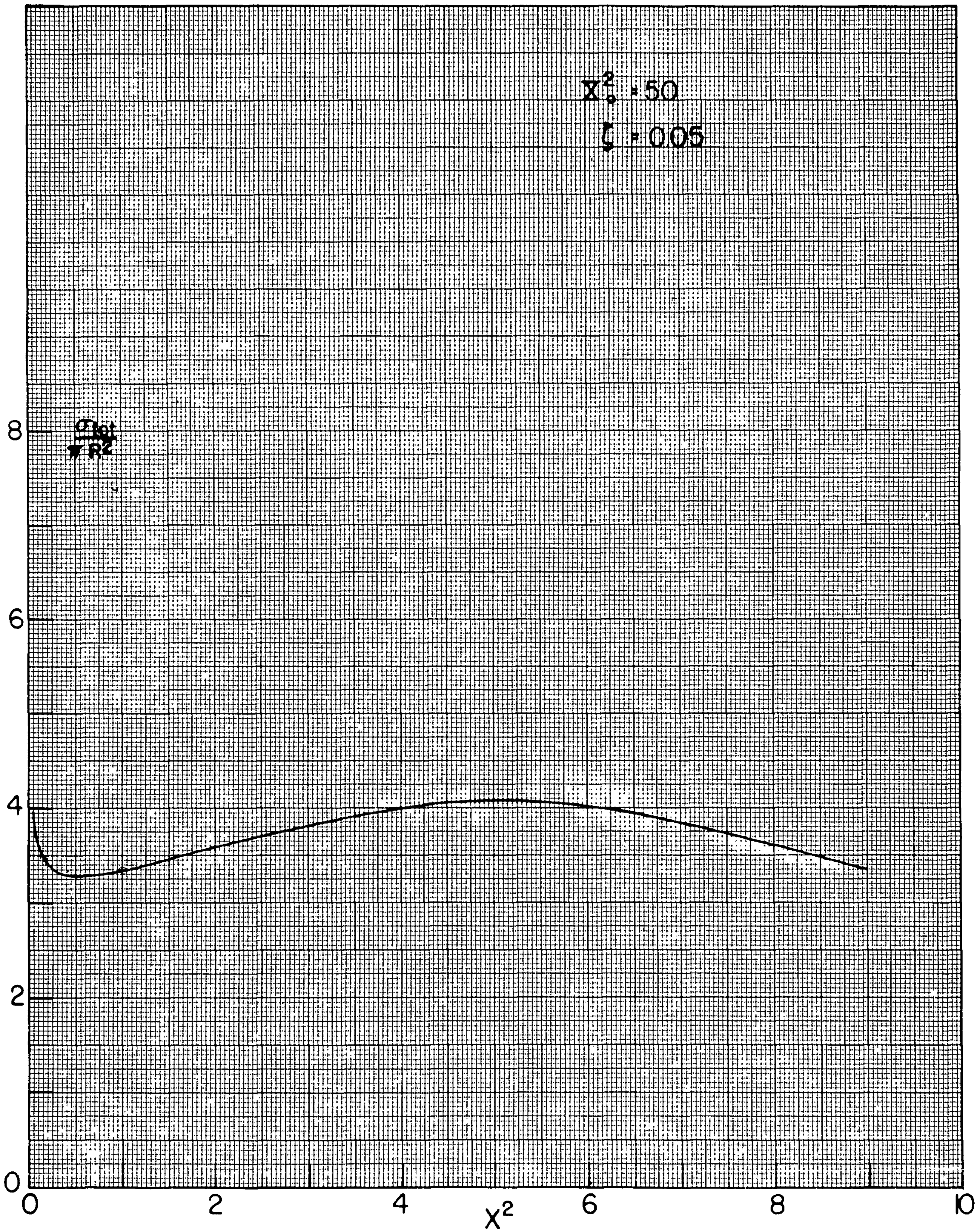
8
6
4
2
0
0
2
4
6
8
10
X^2


$X_o^2 = 50$
$\zeta = 0.05$
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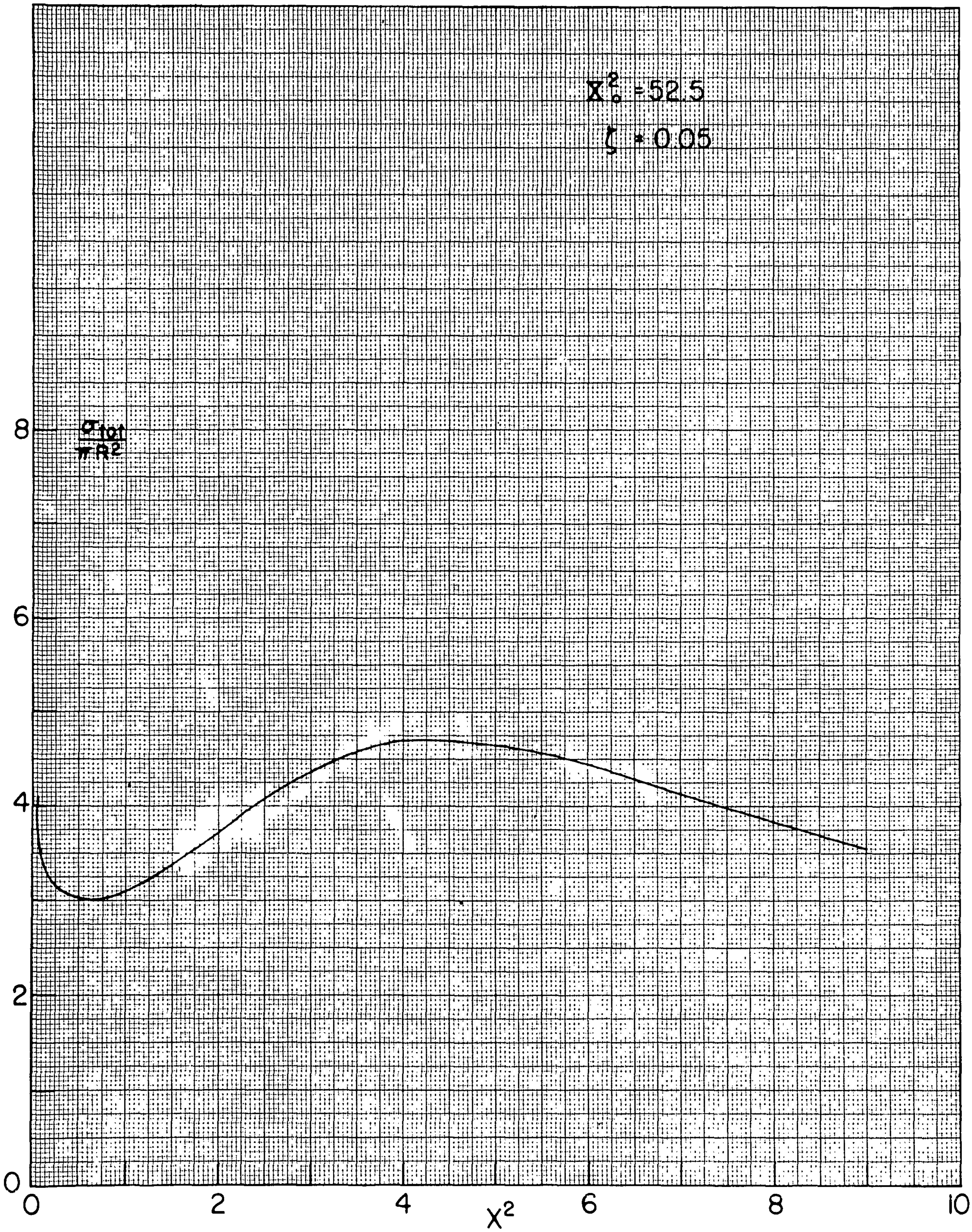
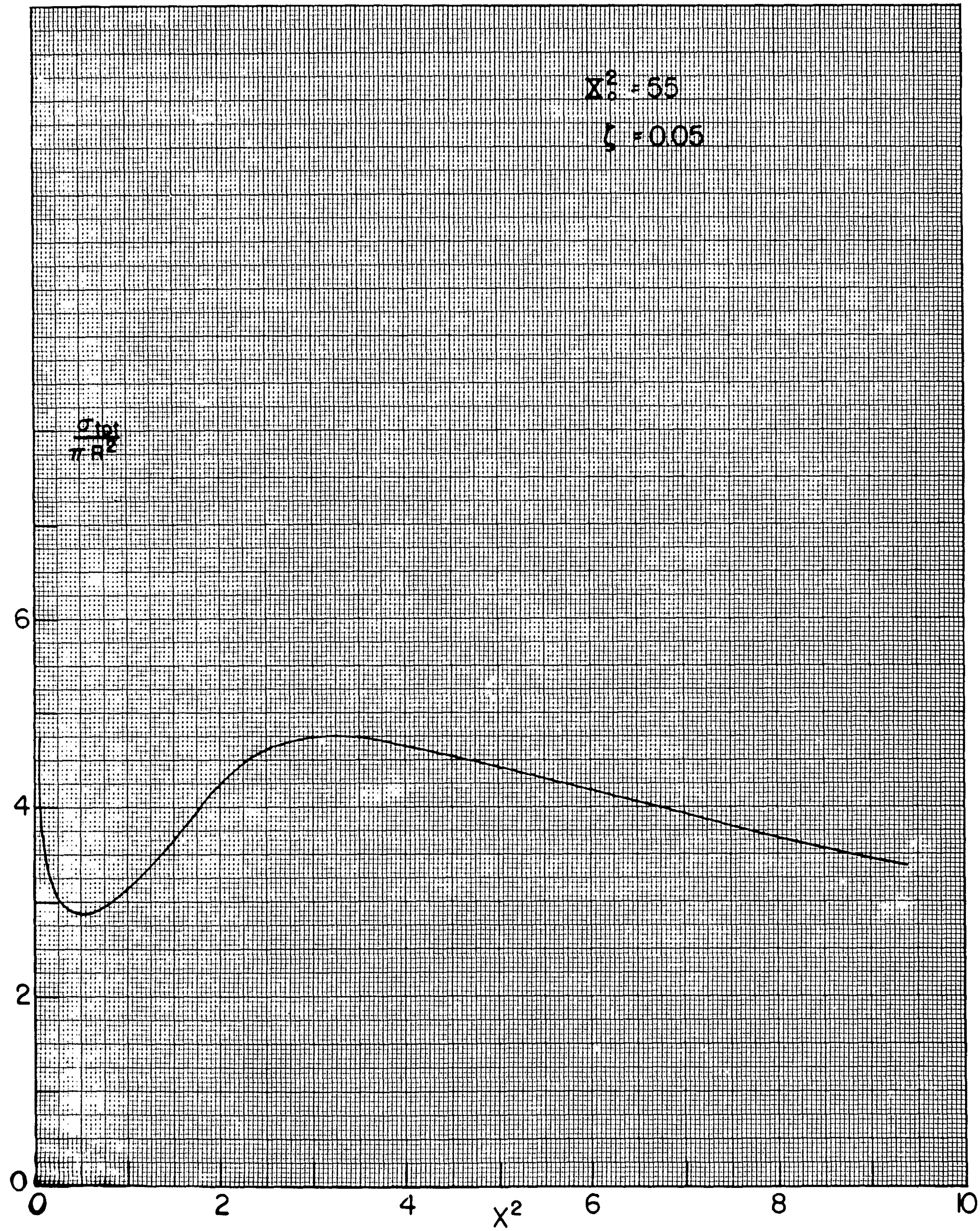
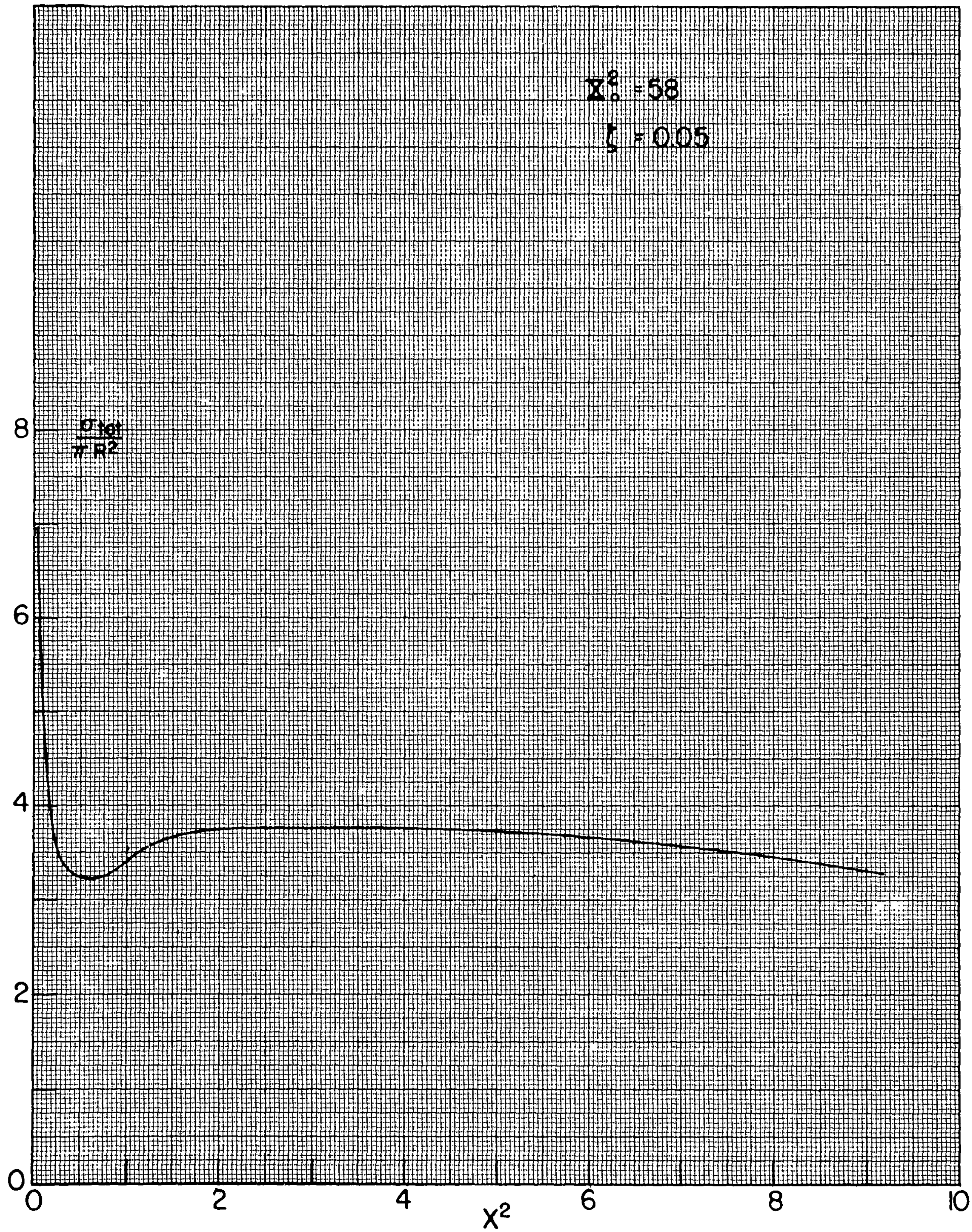
0
2
4
6
8
0
2
4
6
8
10
X^2

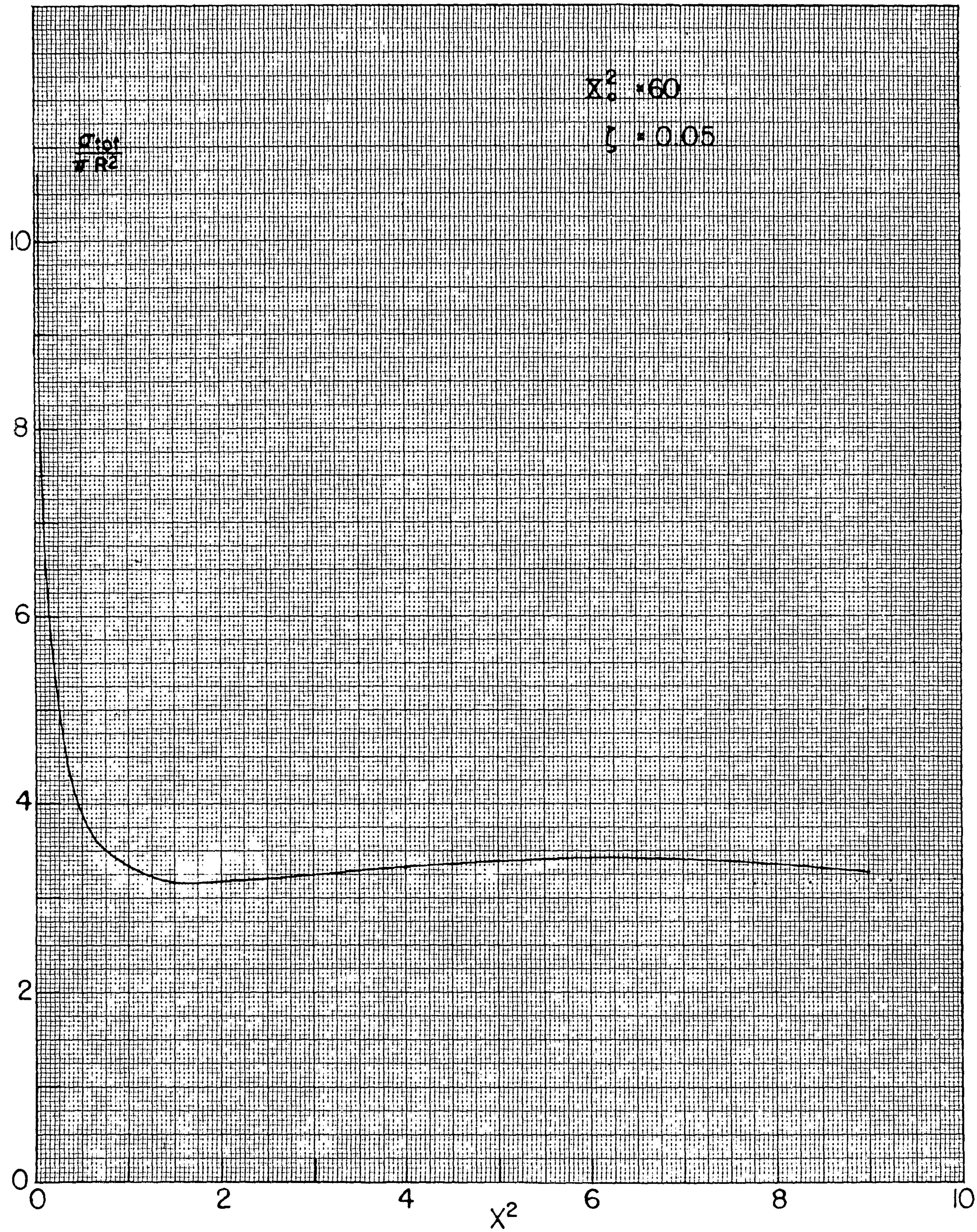


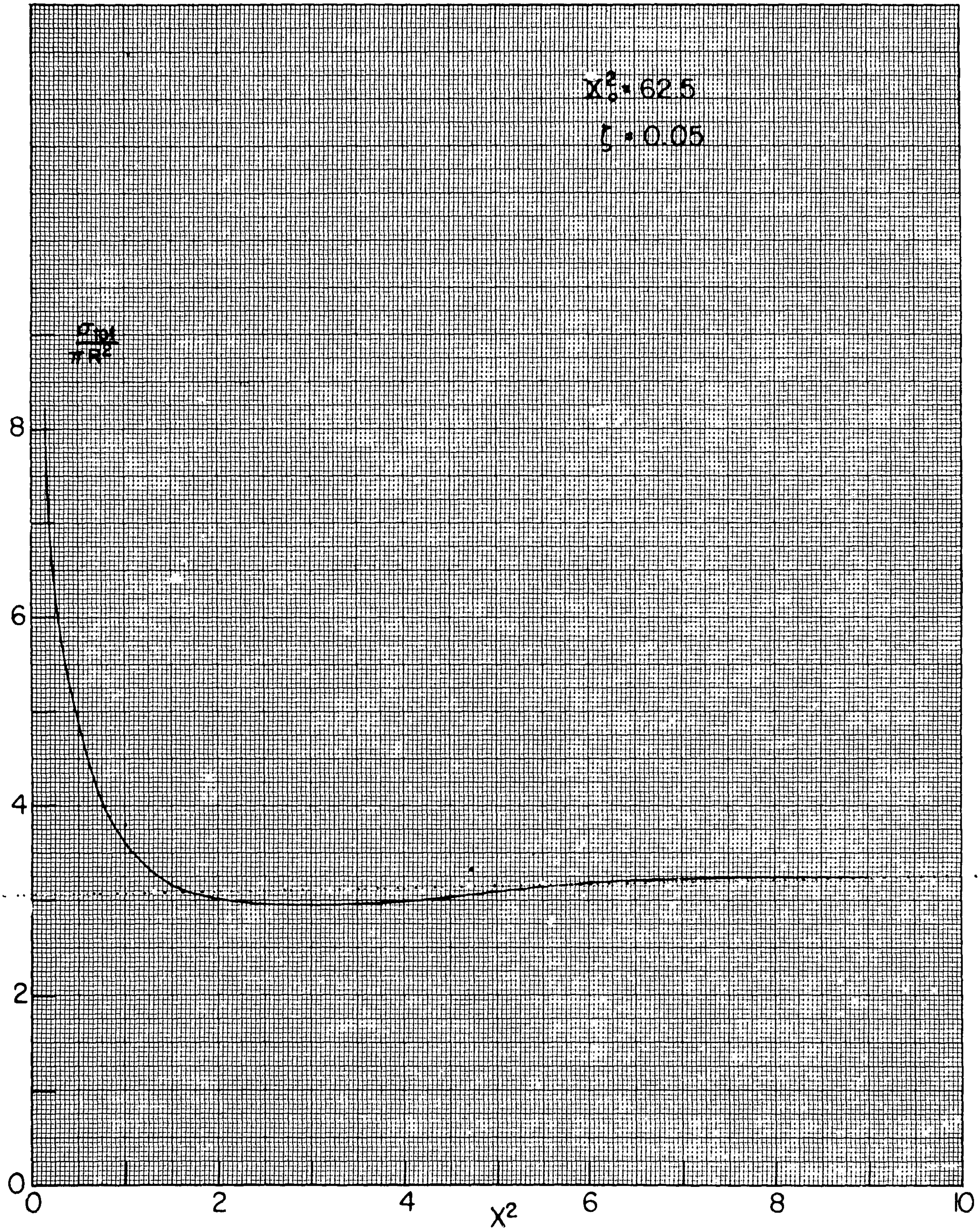
$X_0^2 = 52.5$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
0
2
4
6
8
10
X^2



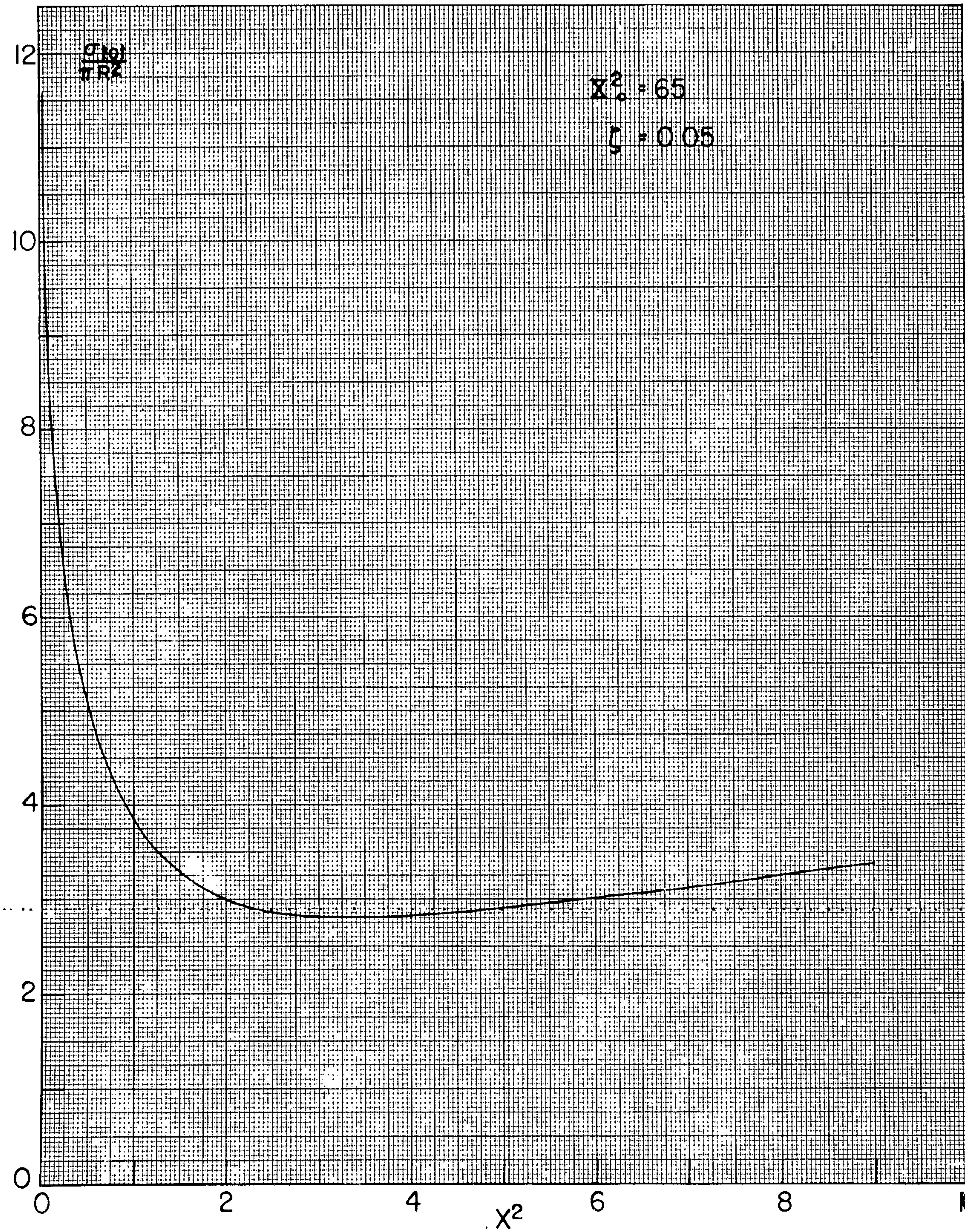
$X_0^2 = 55$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
0
2
4
X^2
6
8
10


$X_0^2 = 58$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
8
6
4
2
0
0
2
4
6
8
10
X^2

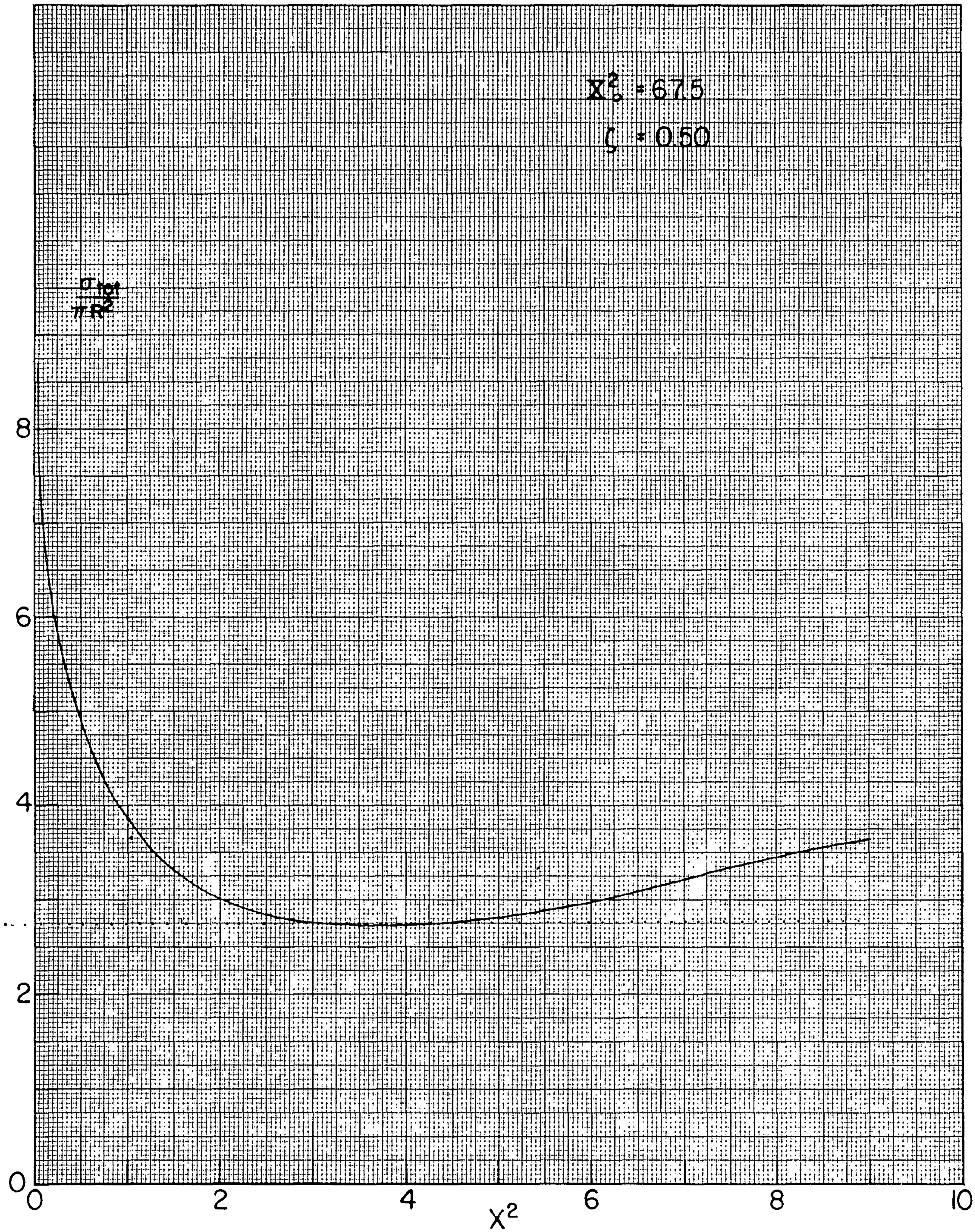

$\frac{\sigma_{tot}}{\pi R^2}$
$X_0^2 = 60$
$\zeta = 0.05$
0
2
4
6
8
10
X^2

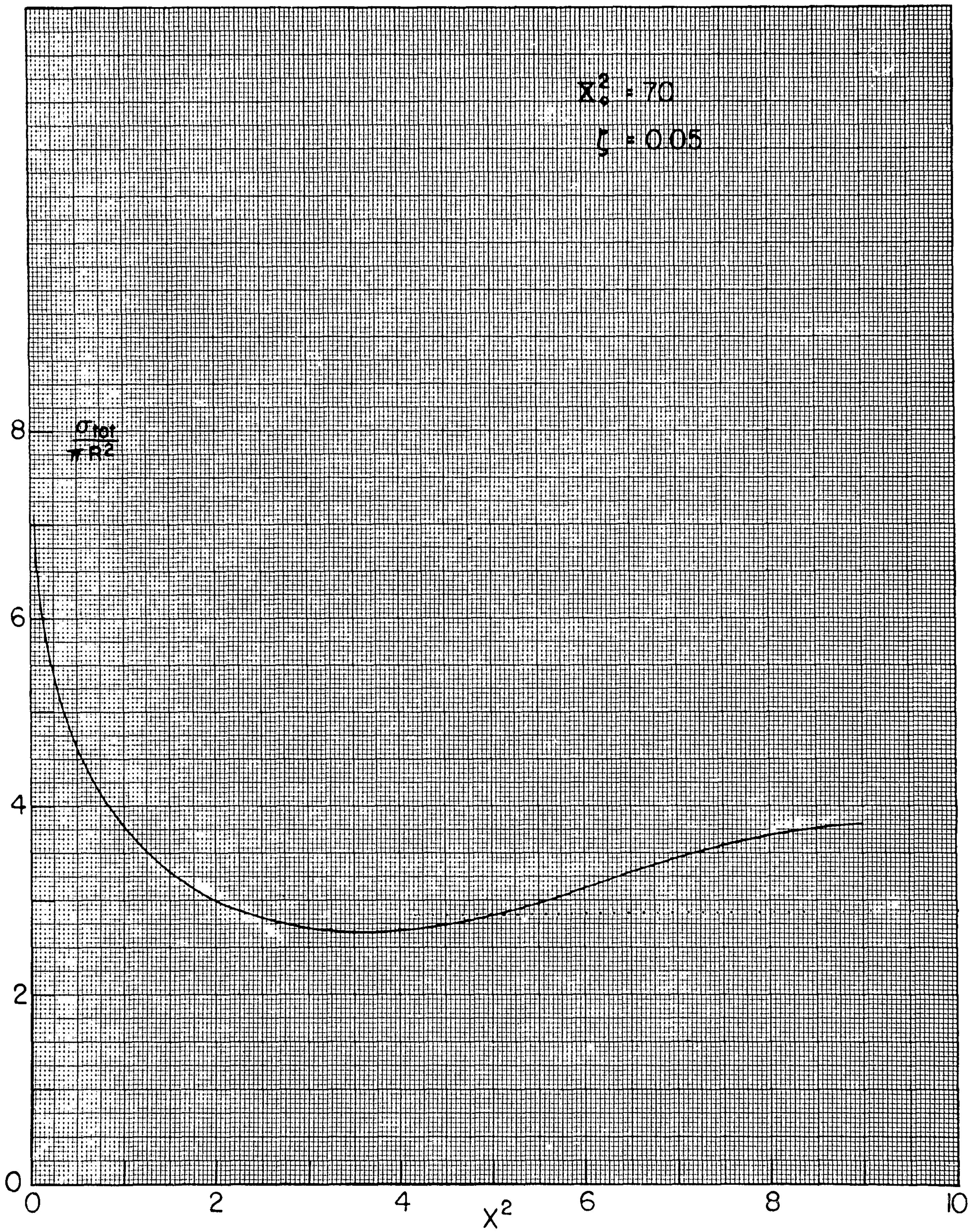


$X_0^2 = 62.5$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
0
2
4
6
8
10
X^2

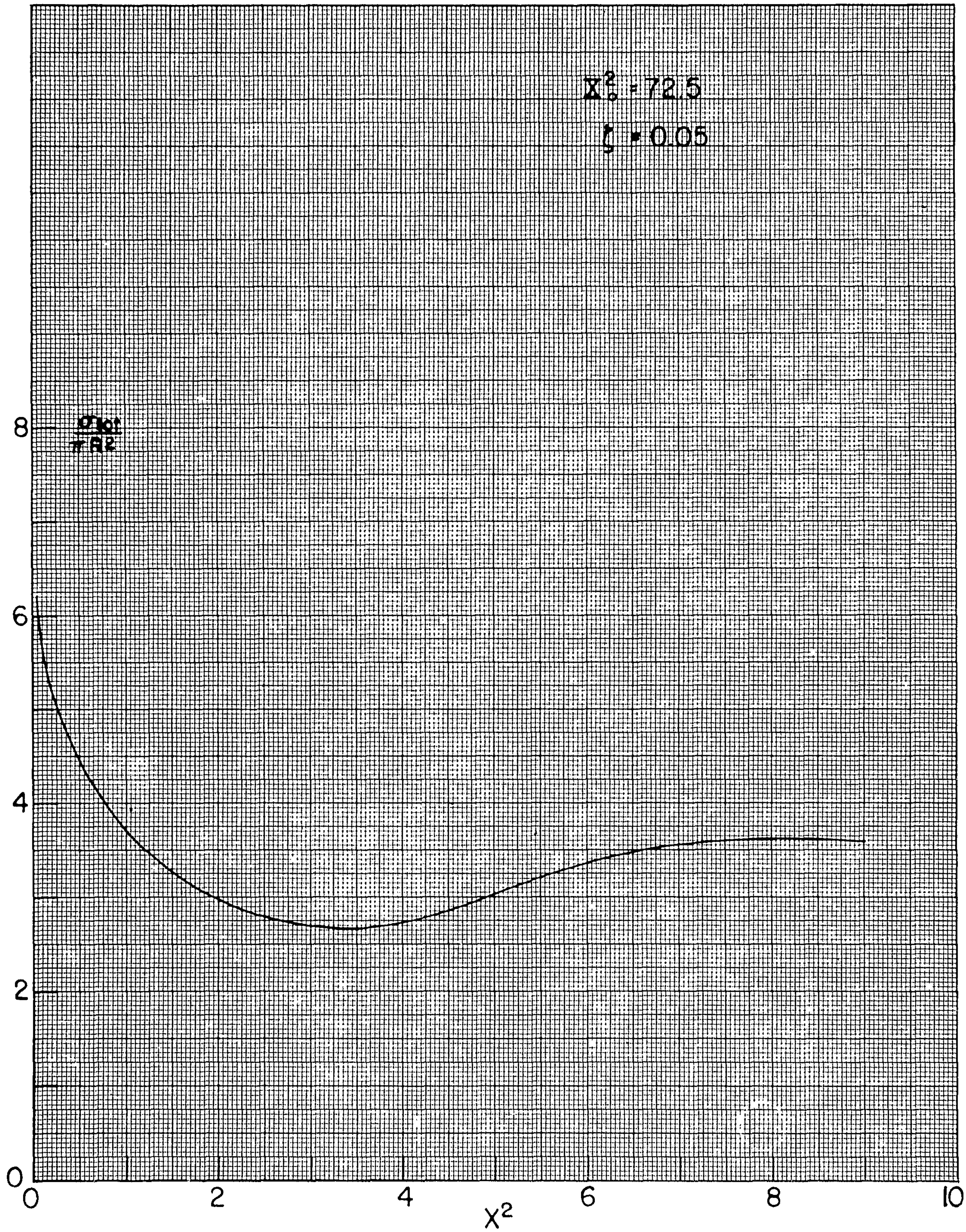


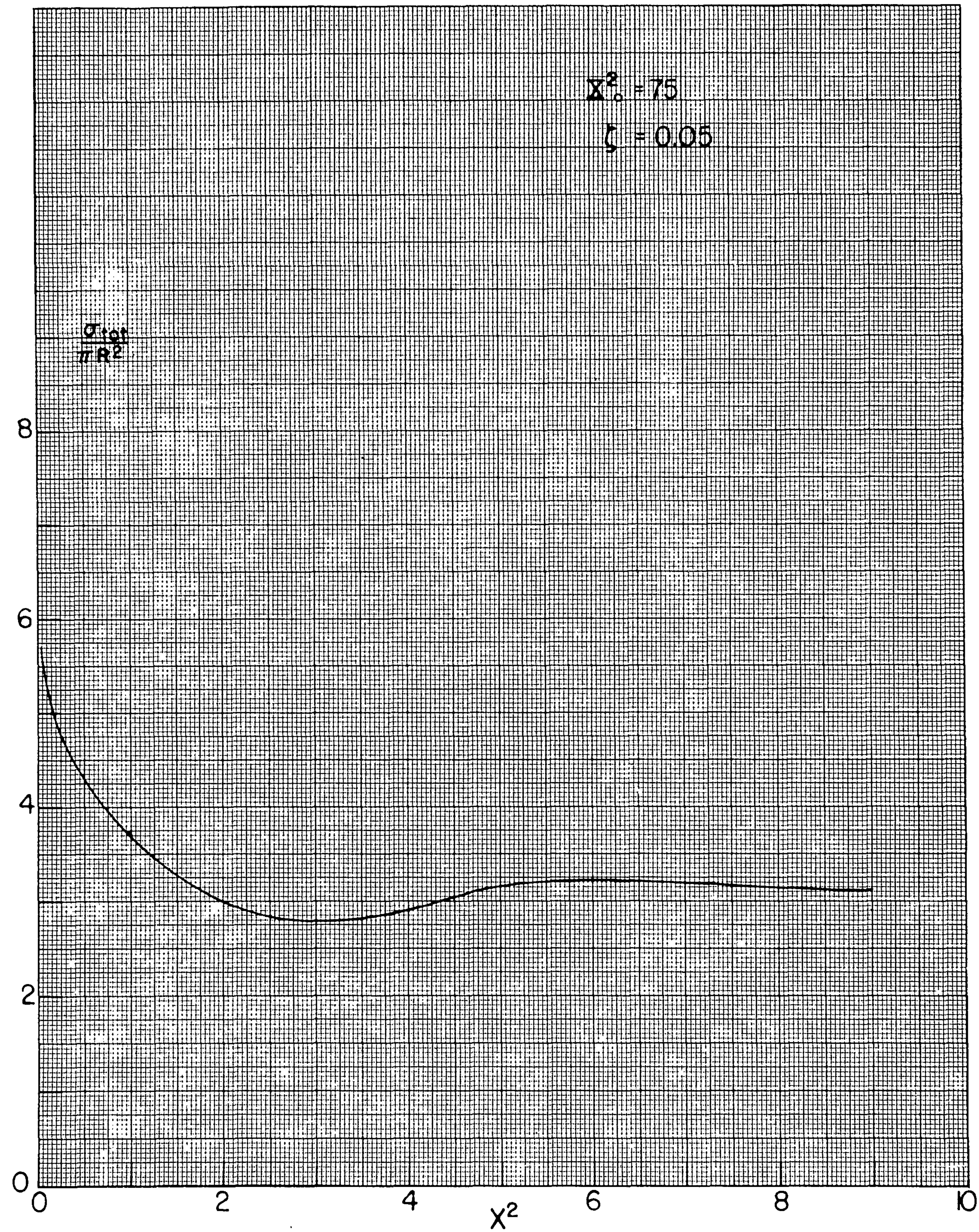
$\frac{\sigma(0)}{\pi R^2}$
$X_0^2 = 65$
$\zeta = 0.05$
0
2
4
6
8
10
12
X^2

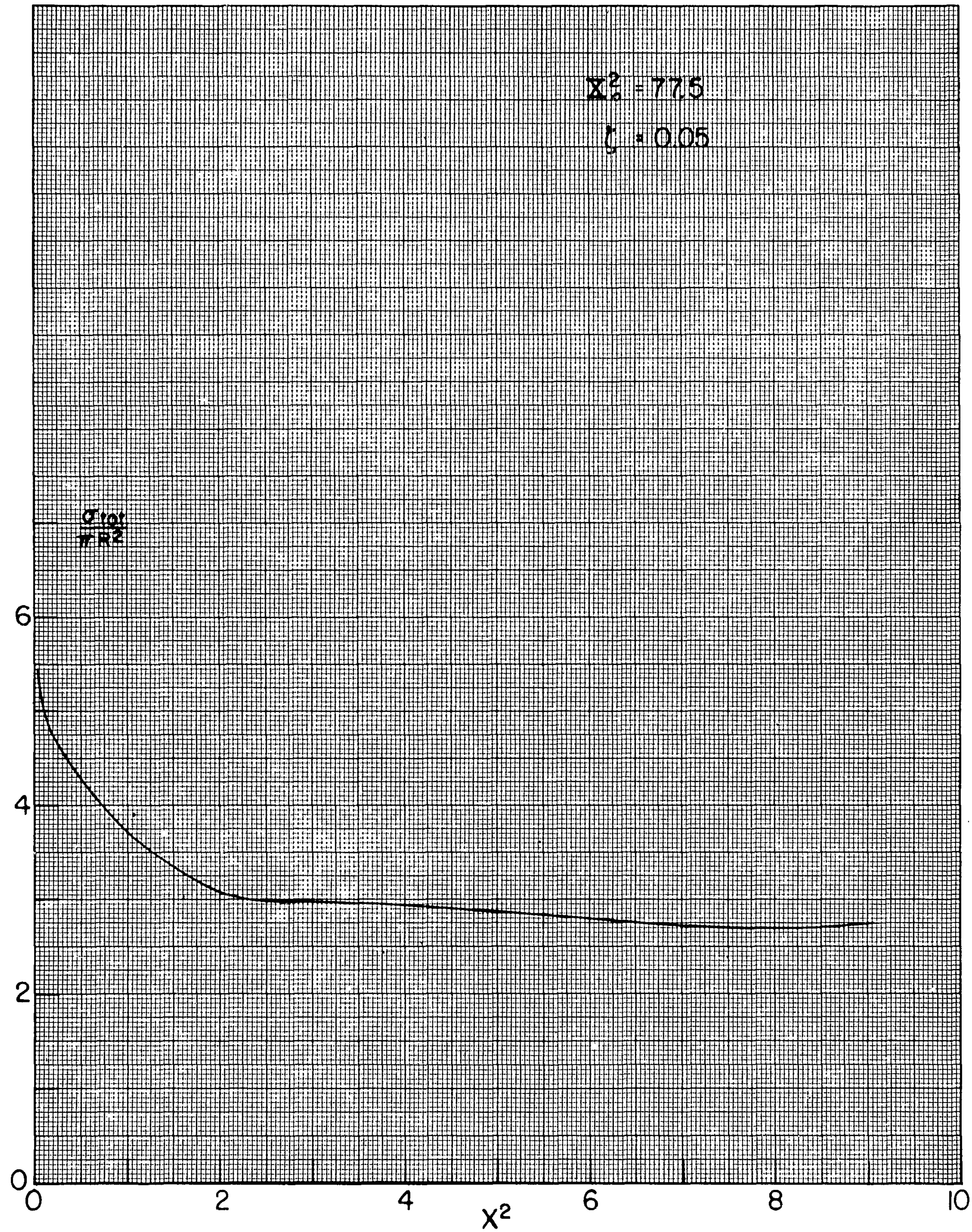

$X_0^2 = 67.5$
$\zeta = 0.50$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
0
2
4
6
8
10
X^2

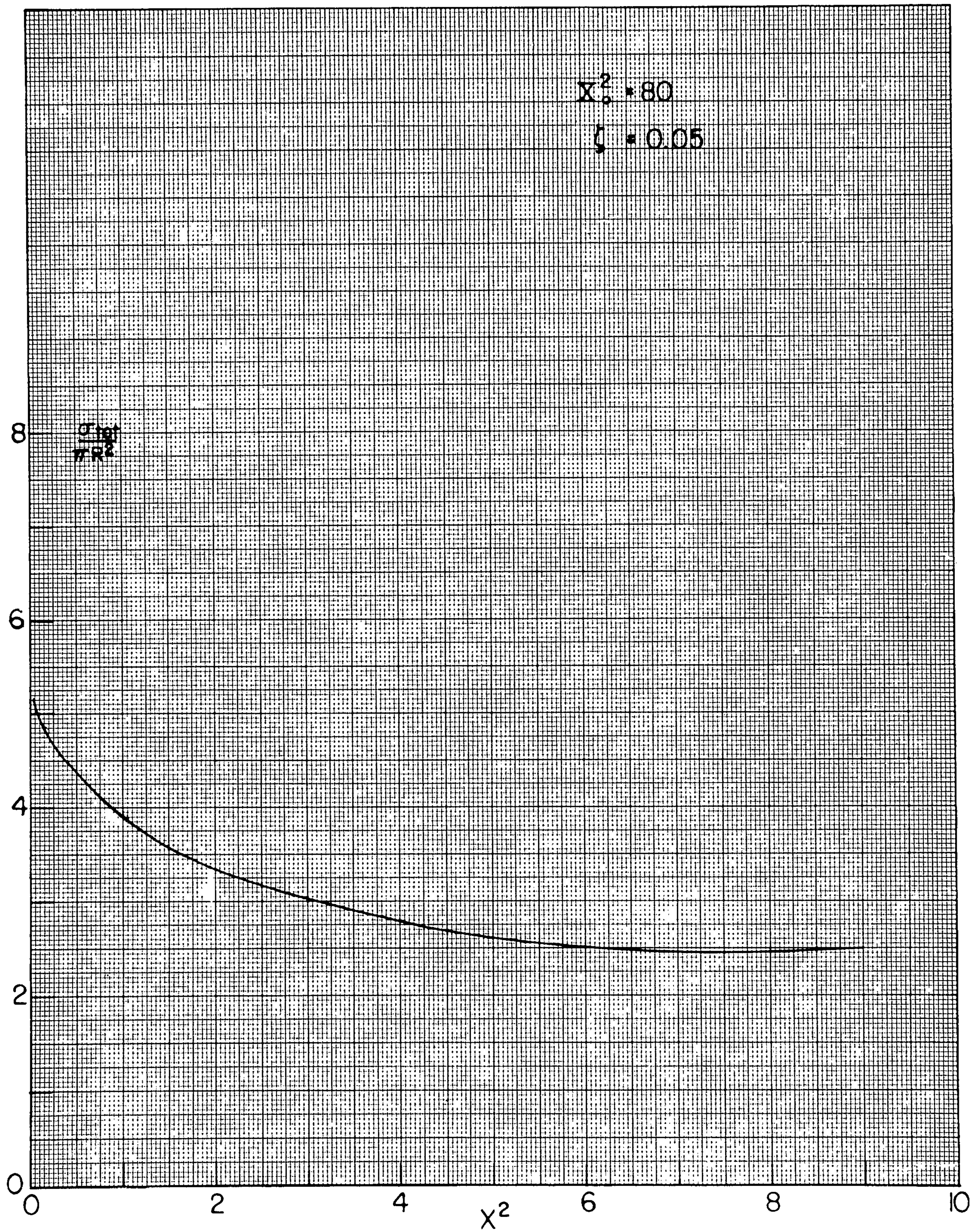


$X_o^2 = 70$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
8
6
4
2
0
0
2
4
6
8
10
X^2


$X_0^2 = 72.5$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
0
2
4
6
8
10
X^2

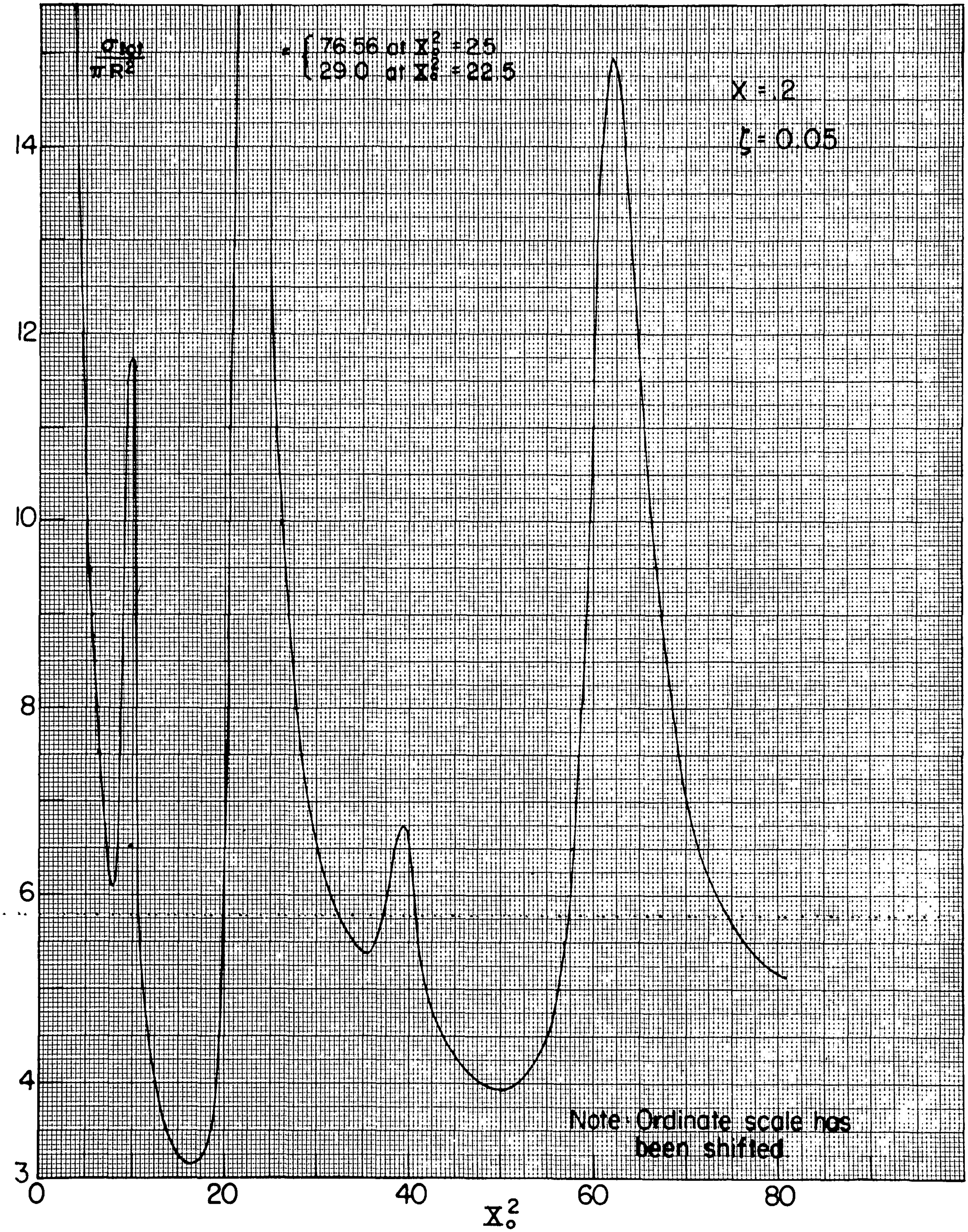

$X_0^2 = 75$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
0
2
4
6
8
10
X^2


$X_0^2 = 77.5$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
10
X^2

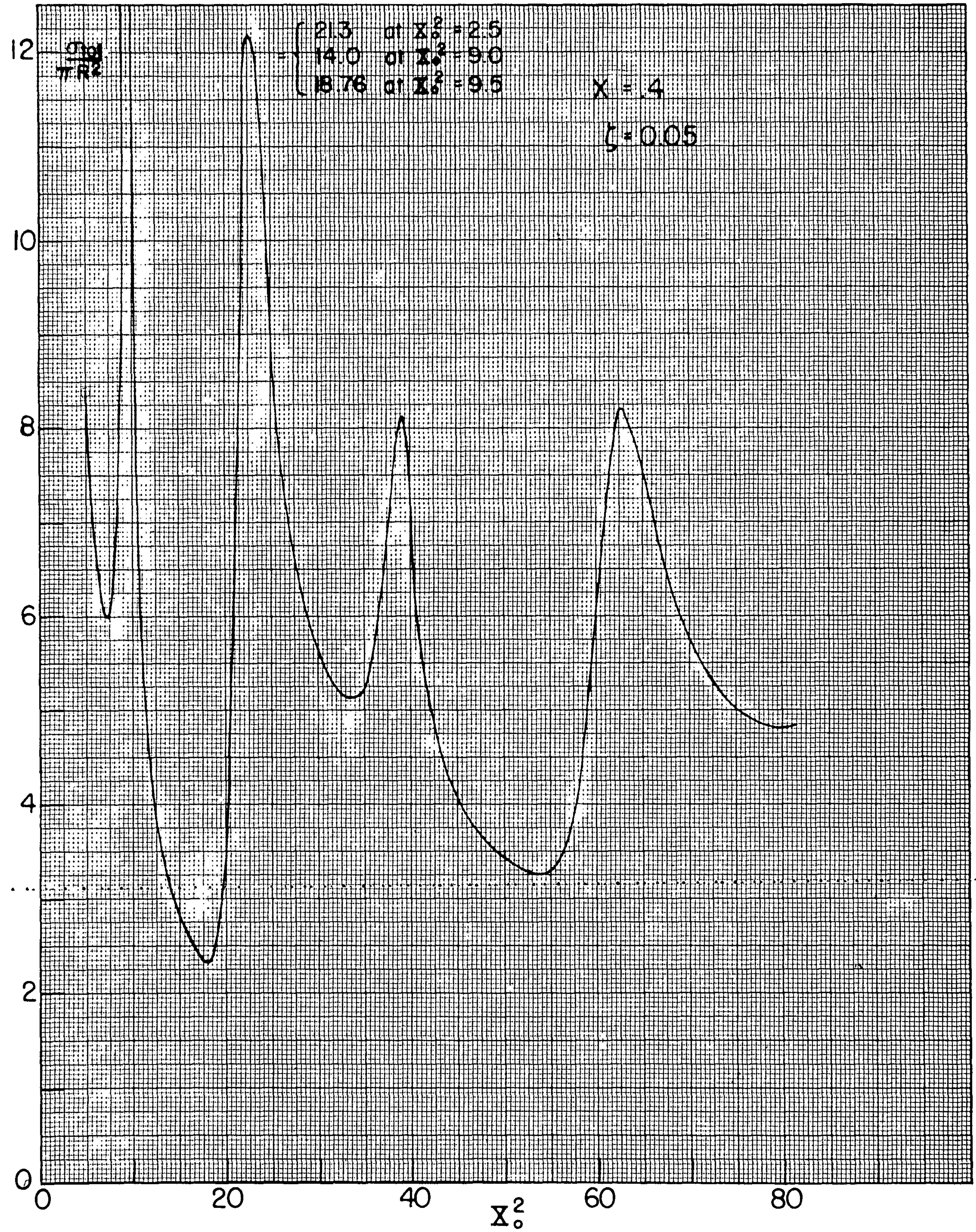


$X_0^2 = 80$
$\zeta = 0.05$
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
10
X^2

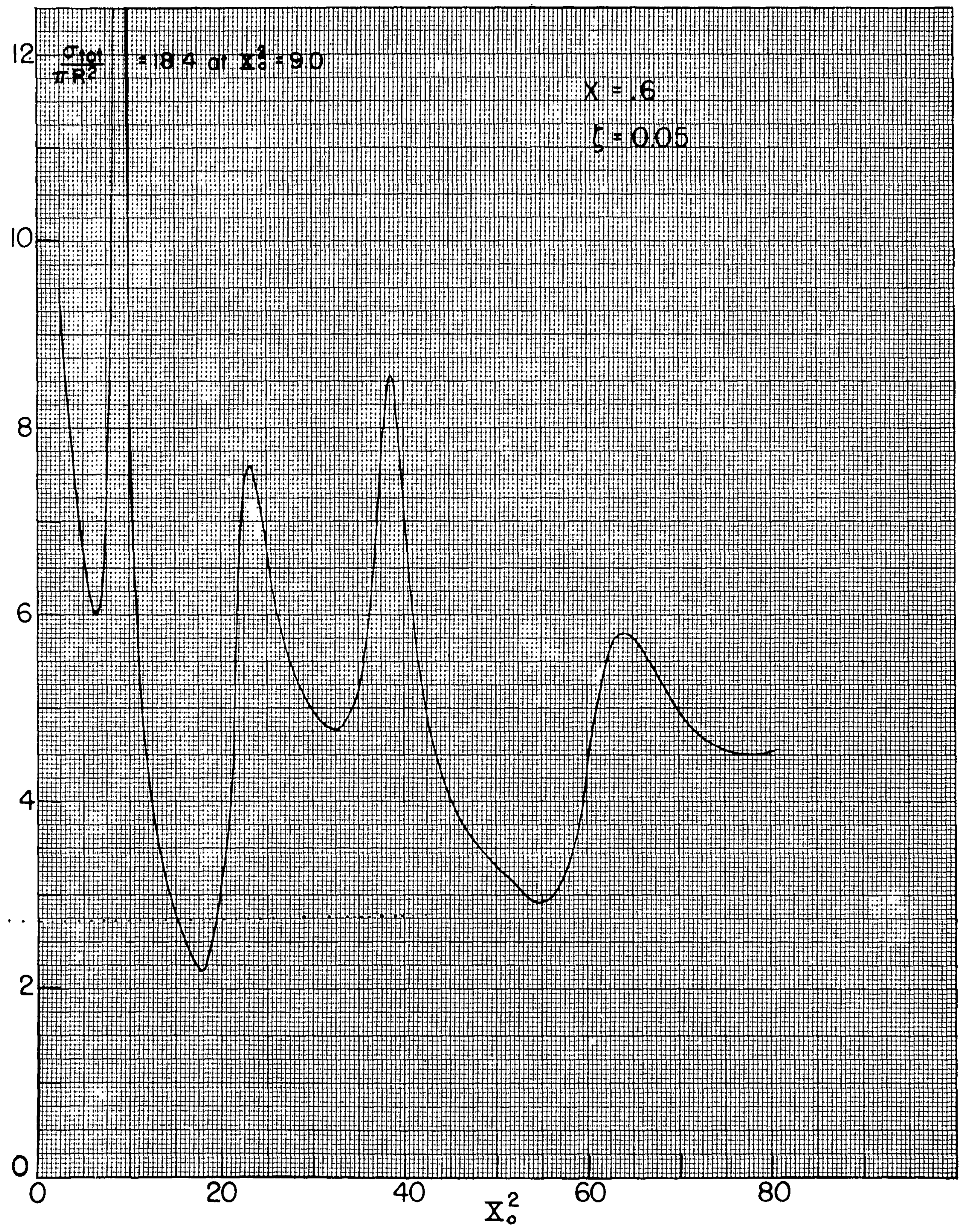
III. PLOTS OF TOTAL CROSS SECTION VS. X_o^2 FOR FIXED x^2 WITH $\zeta = 0.05$

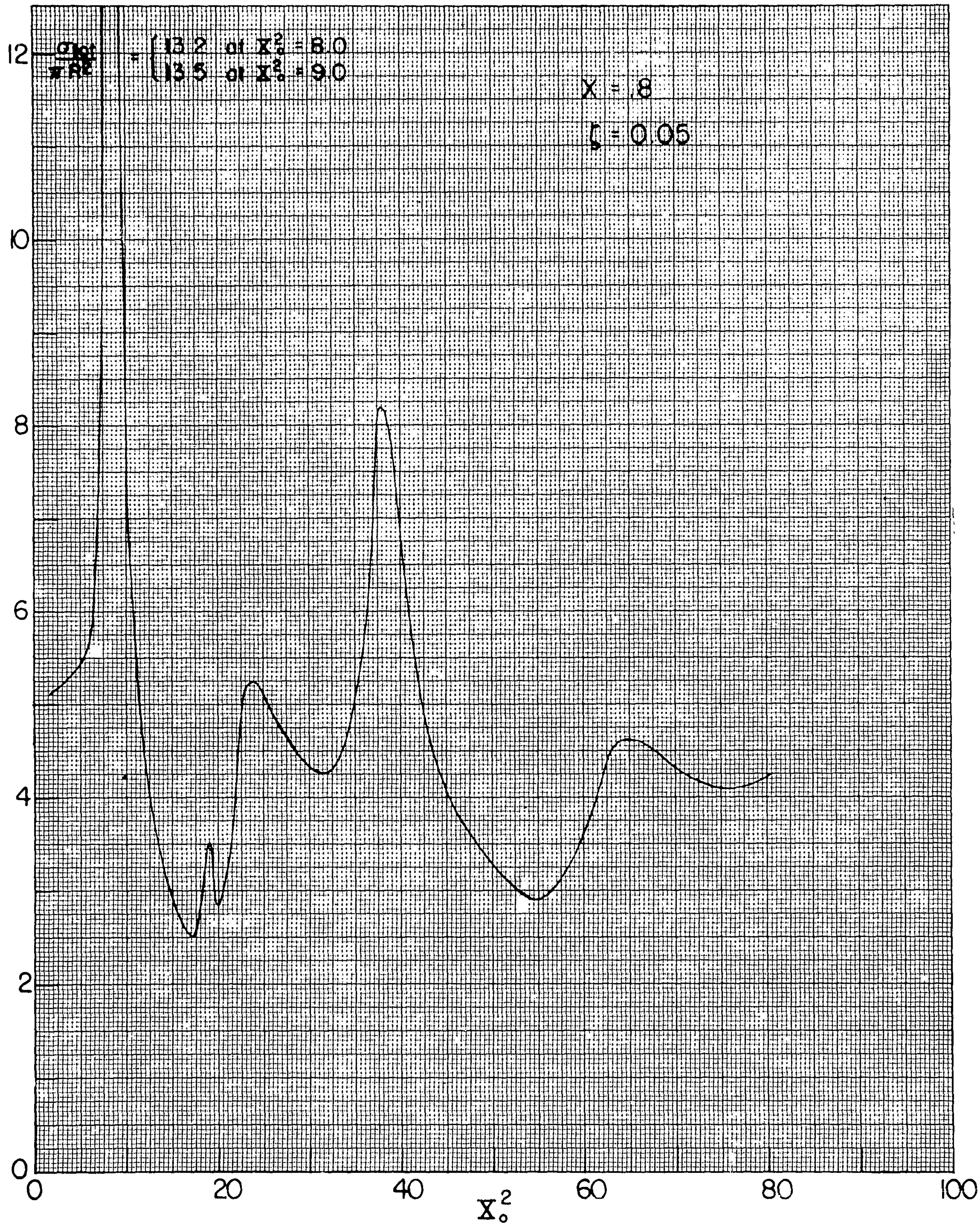


$\frac{\sigma_{tot}}{\pi R^2}$
$\begin{cases} 76.56 \text{ at } X_o^2 = 25 \\ 29.0 \text{ at } X_o^2 = 22.5 \end{cases}$
X = .2
ζ = 0.05
14
12
10
8
6
4
3
0
20
40
60
80
X_o^2
Note: Ordinate scale has been shifted

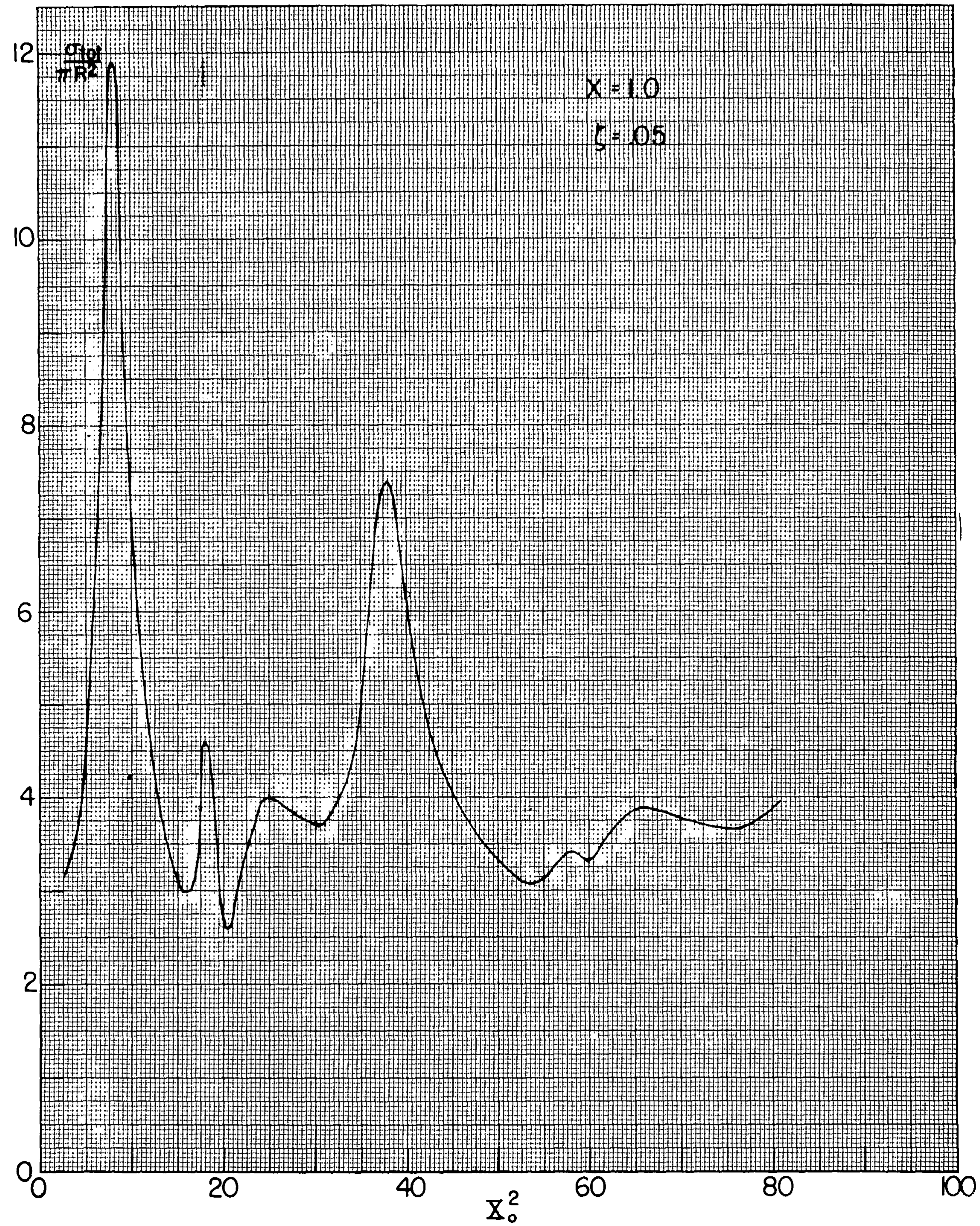


σtot / πR²
= 21.3 at X₀² = 2.5
14.0 at X₀² = 9.0
18.76 at X₀² = 9.5
X = .4
ζ = 0.05
12
10
8
6
4
2
0
0
20
40
60
80
X₀²

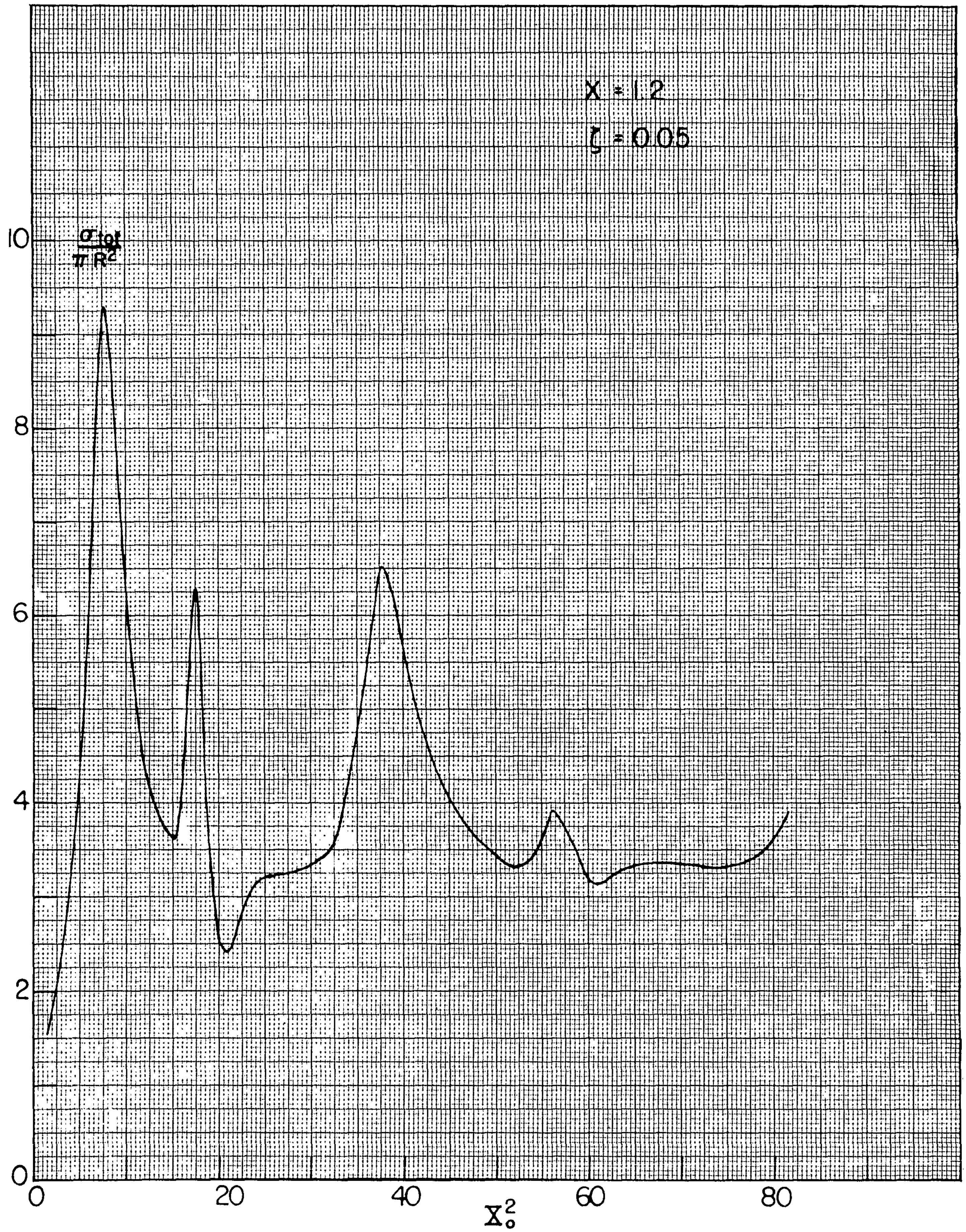

$\frac{\sigma_{tot}}{\pi R^2}$ = 18.4 at X_o^2 = 9.0
X = .6
ζ = 0.05
0
2
4
6
8
10
12
0
20
40
60
80
X_o^2



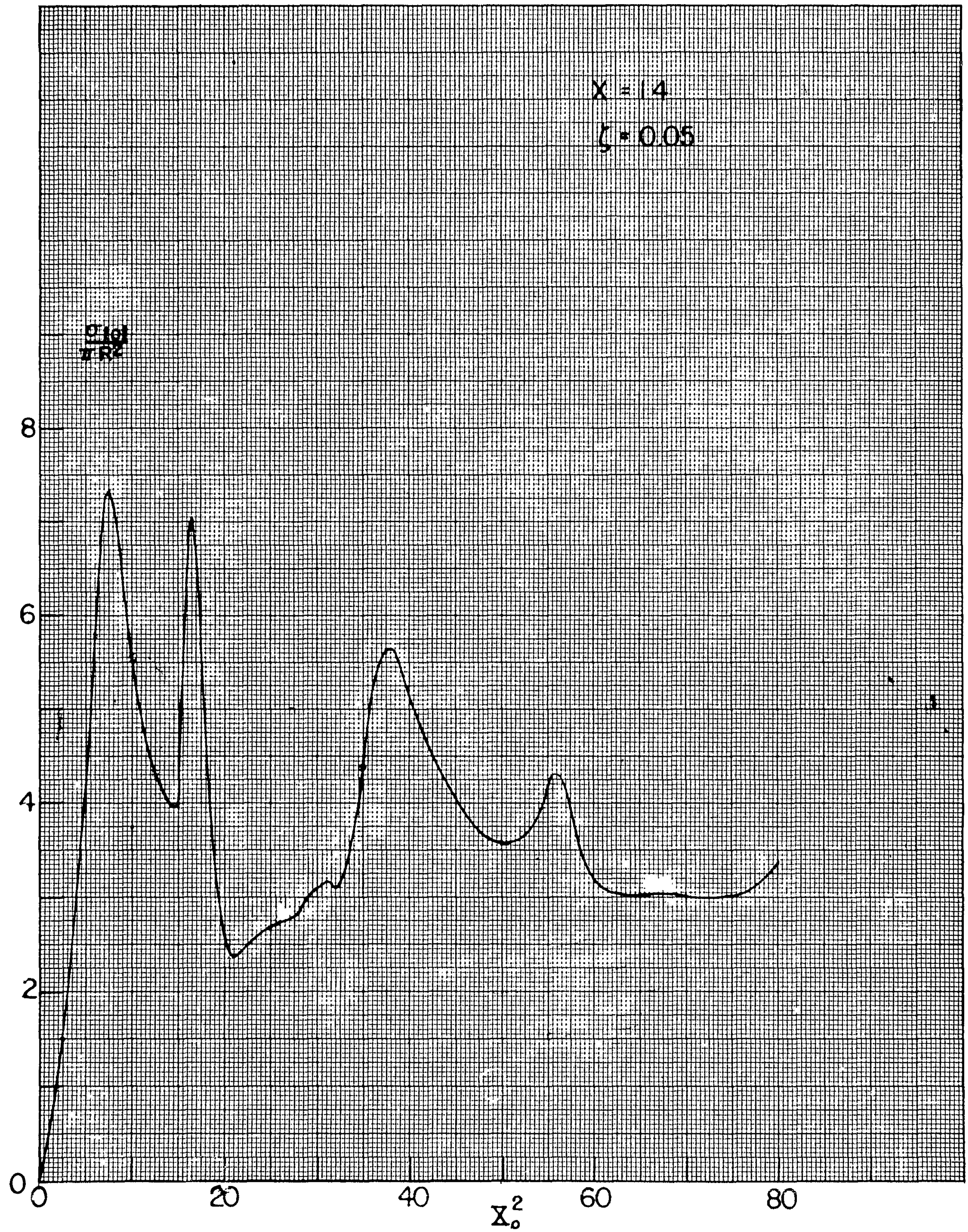
σ_tot / πR² = { 13.2 at X₀² = 8.0 ; 13.5 at X₀² = 9.0 }
X = .8
ζ = 0.05
12
10
8
6
4
2
0
0
20
40
60
80
100
X₀²

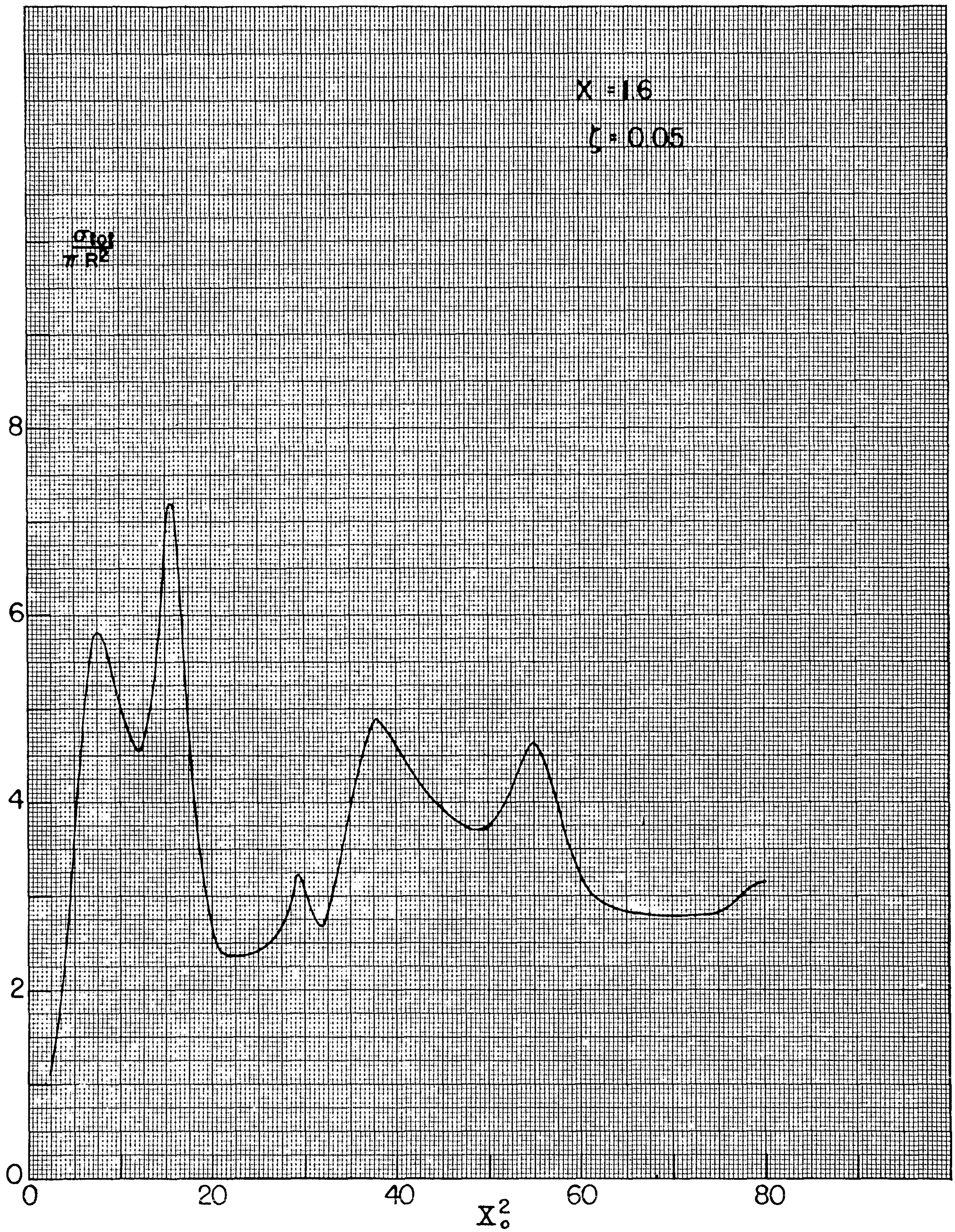


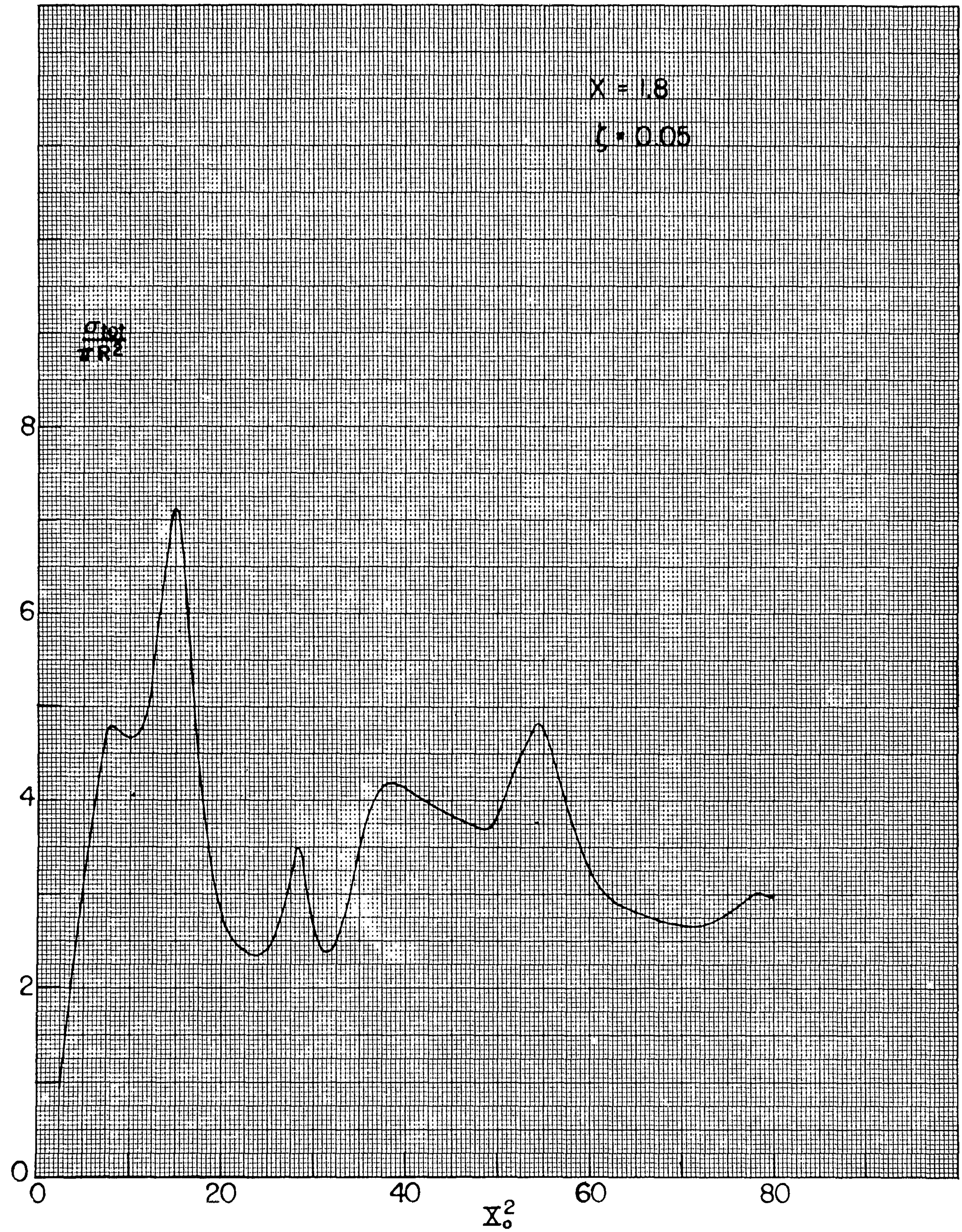
$\frac{\sigma_{tot}}{\pi R^2}$
X = 1.0
ζ = .05
0
2
4
6
8
10
12
0
20
40
60
80
100
X_o^2

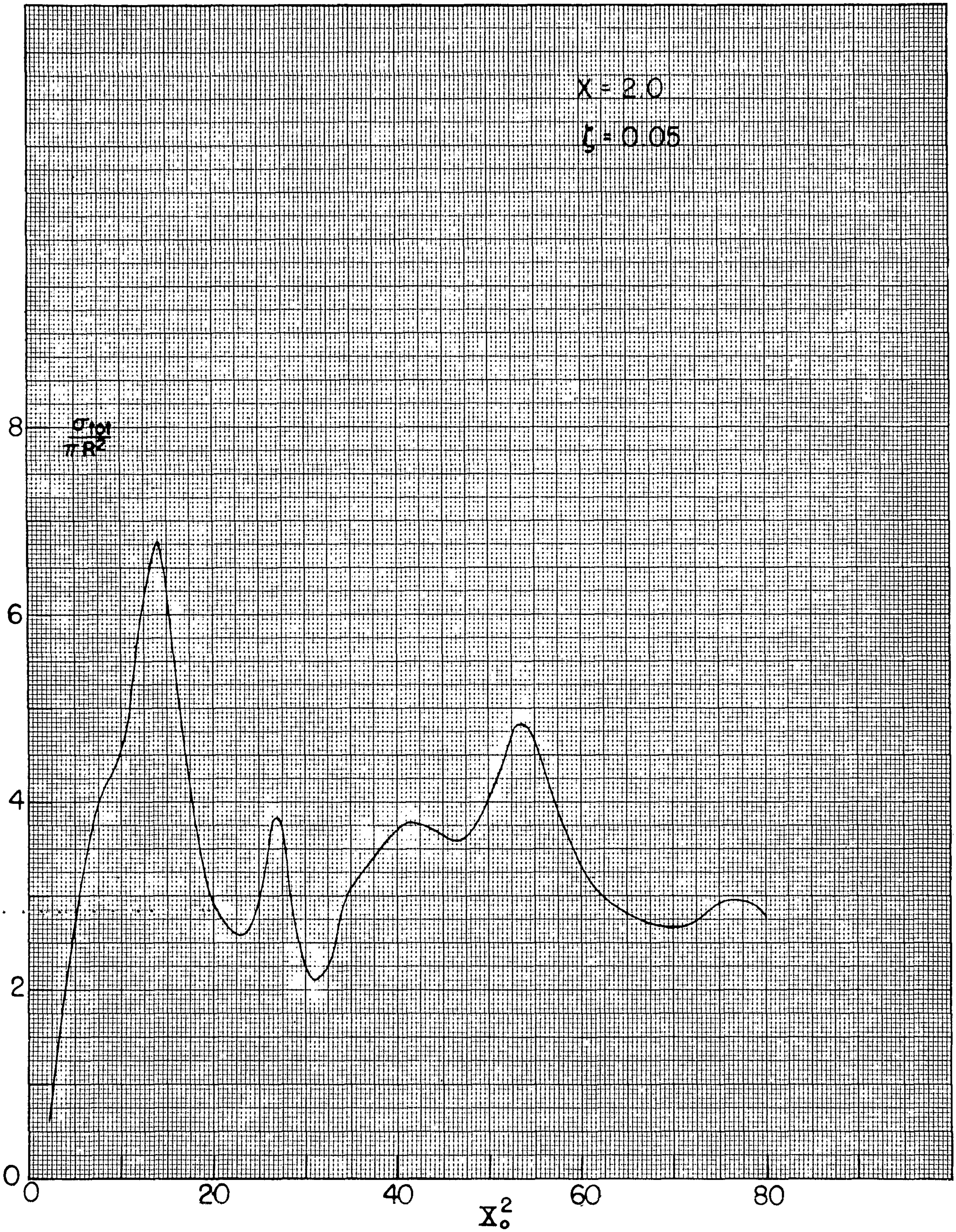


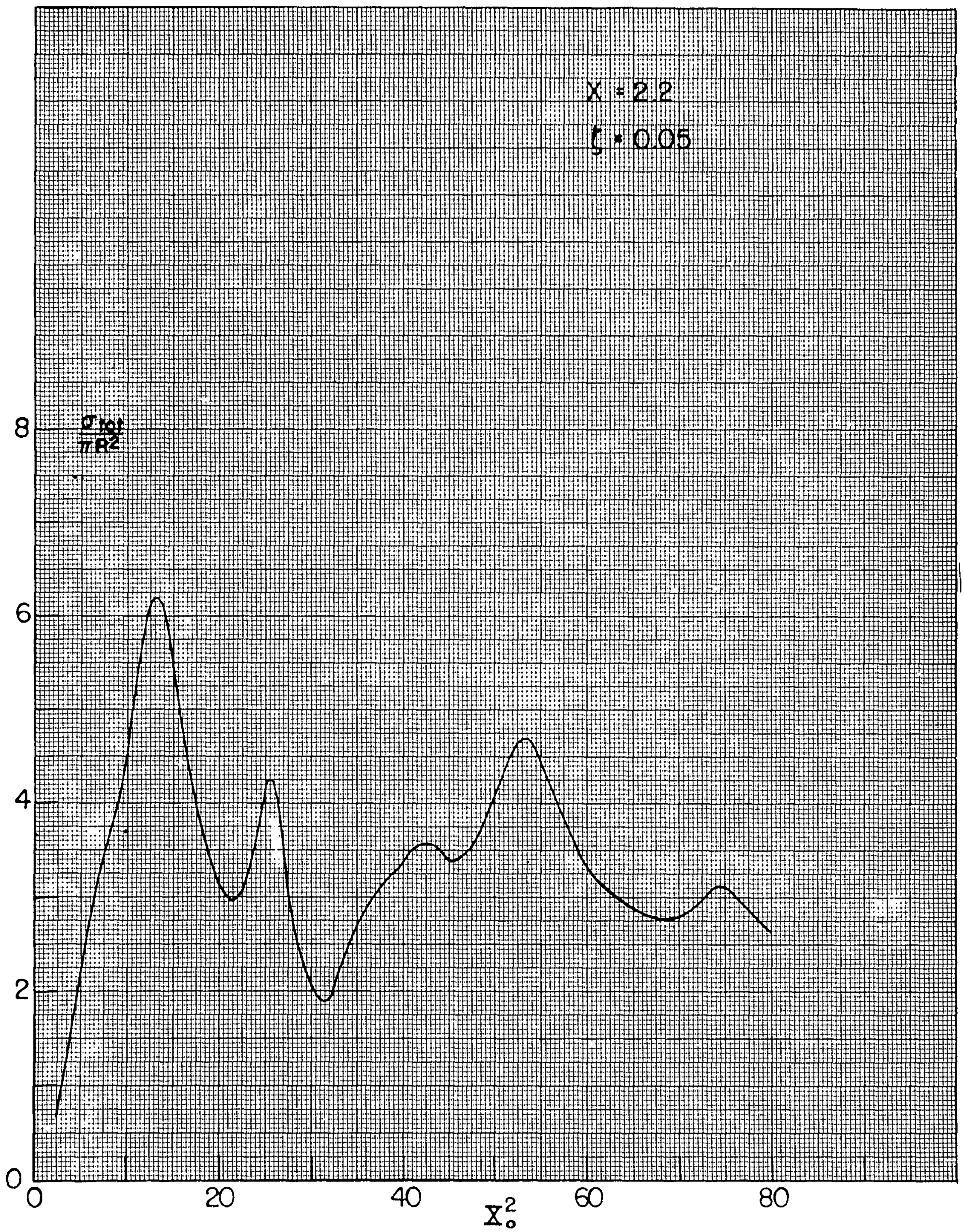
X = 1.2
ζ = 0.05
$\frac{\sigma_{tot}}{\pi R^2}$
10
8
6
4
2
0
0
20
40
60
80
X_o^2

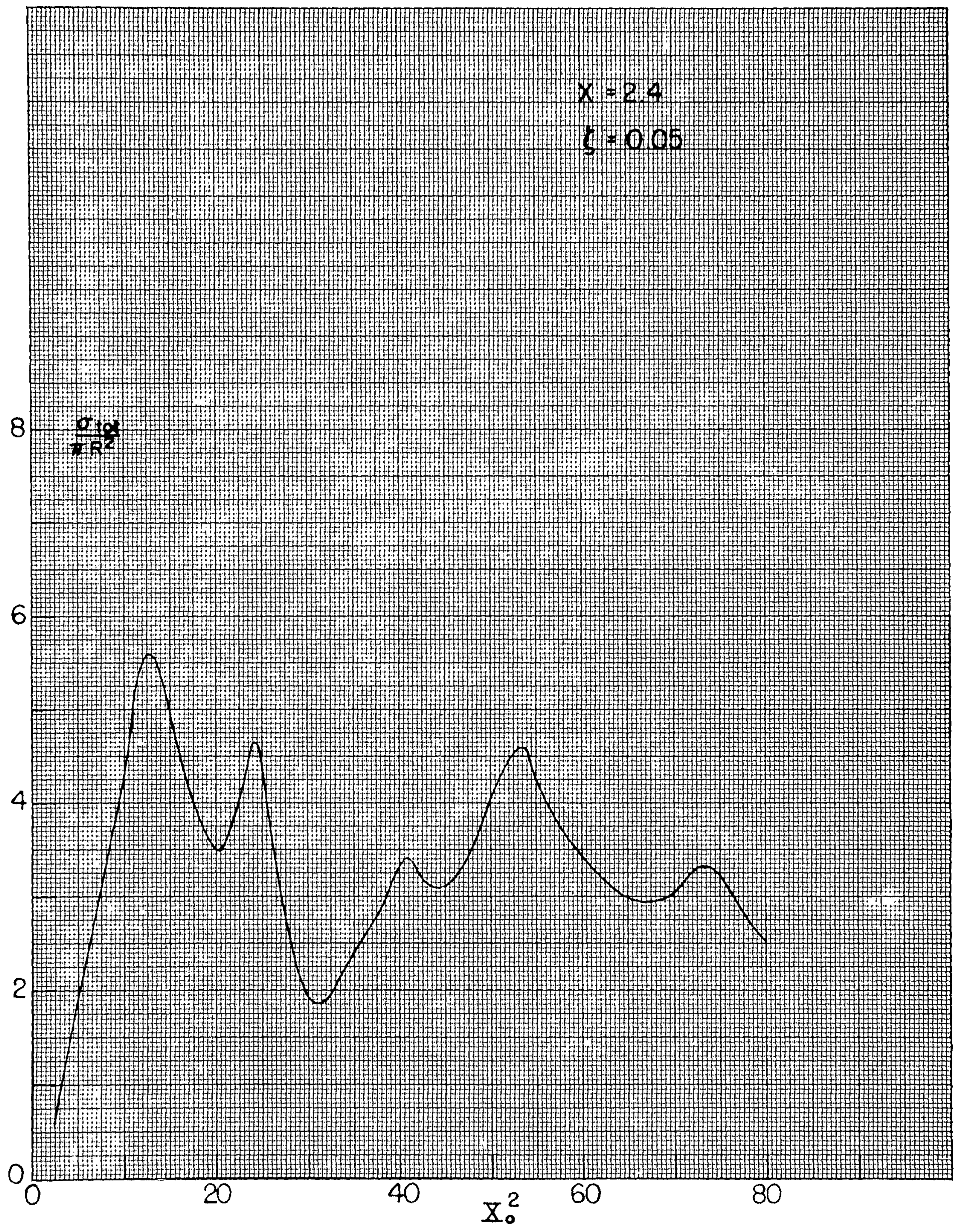

χ = 14
ζ = 0.05
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
0
20
40
60
80
X_o^2


X = 1.6
ζ = 0.05
$\frac{\sigma_{tot}}{\pi R^2}$
8
6
4
2
0
0
20
40
60
80
X_o^2

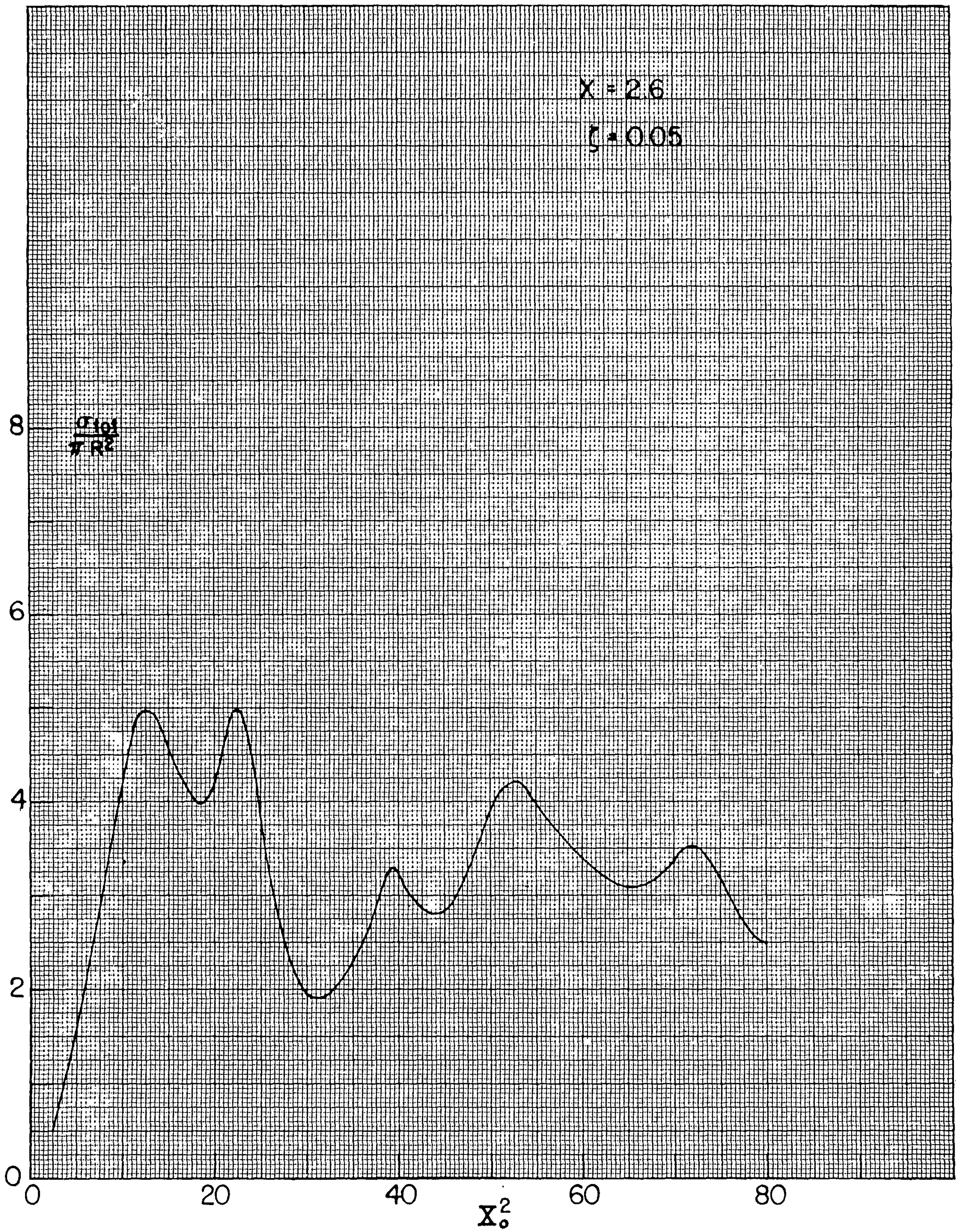

X = 1.8
ζ = 0.05
σ tot / πR²
0
2
4
6
8
0
20
40
60
80
X₀²


X = 2.0
ζ = 0.05
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
8
0
20
40
60
80
X_o^2

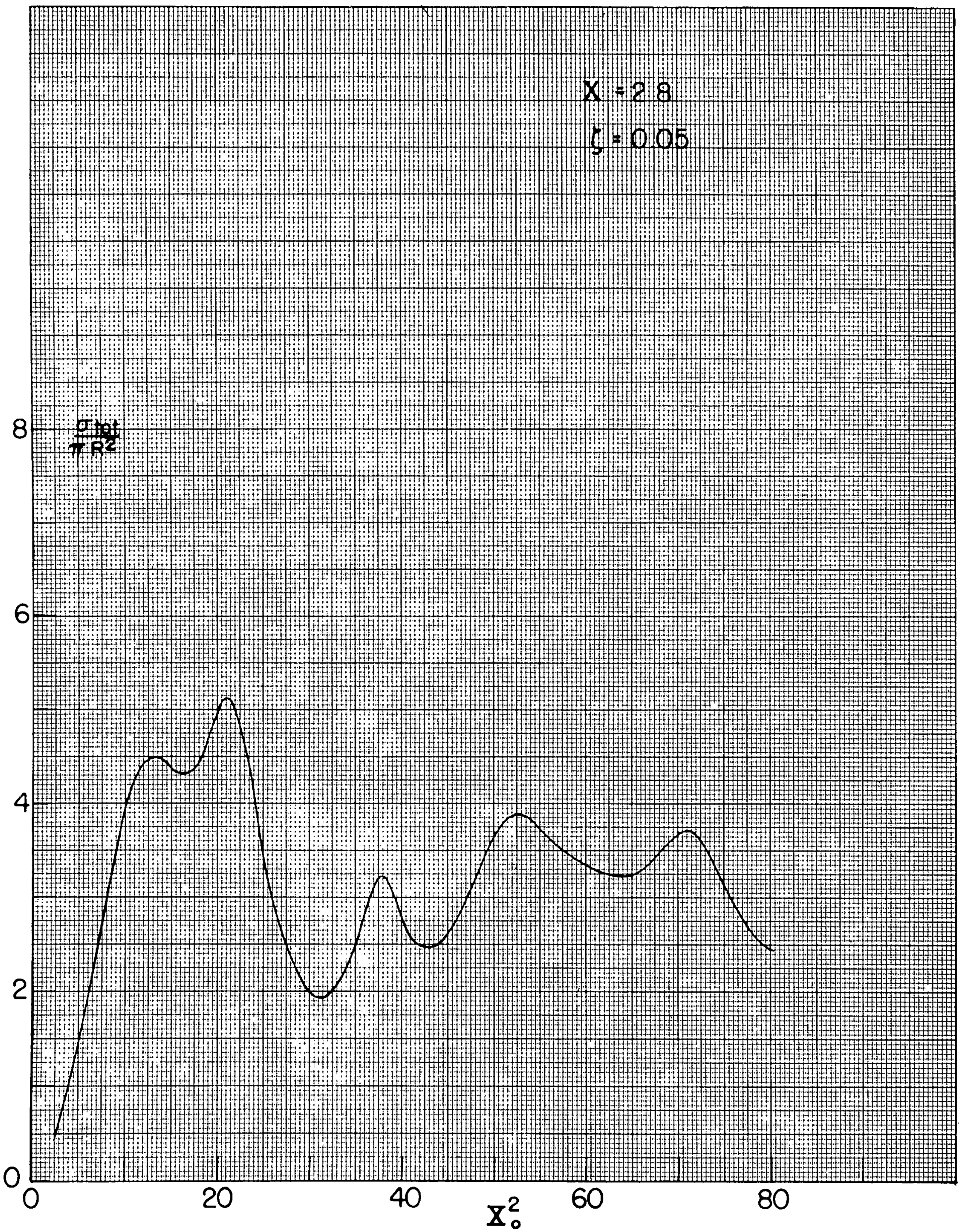

X = 2.2
ζ = 0.05
8
6
4
2
0
$\frac{\sigma_{tot}}{\pi R^2}$
0
20
40
60
80
X_o^2

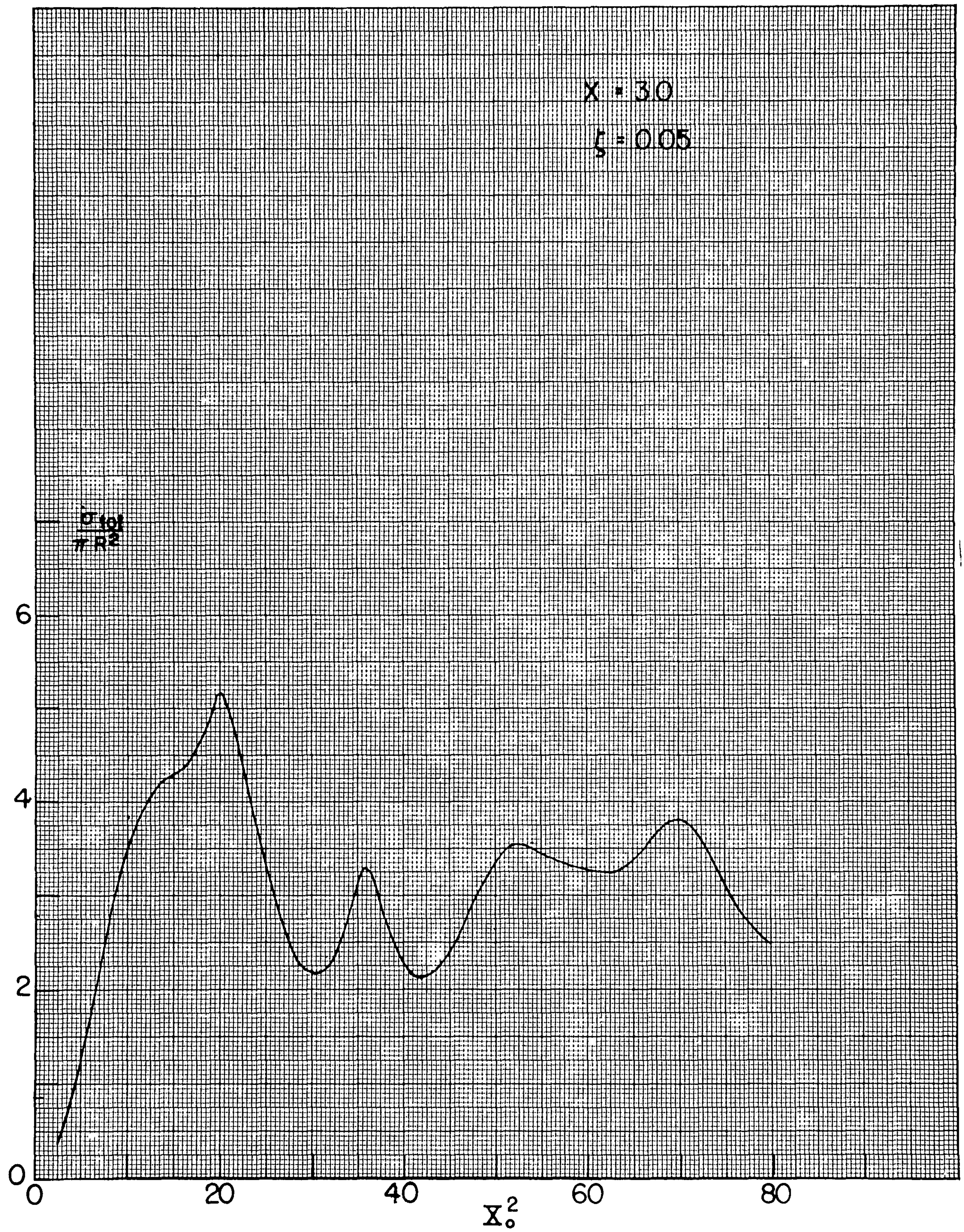


X = 2.4
ζ = 0.05
$\frac{\sigma_{tot}}{\pi R^2}$
8
6
4
2
0
0
20
40
60
80
X_o^2

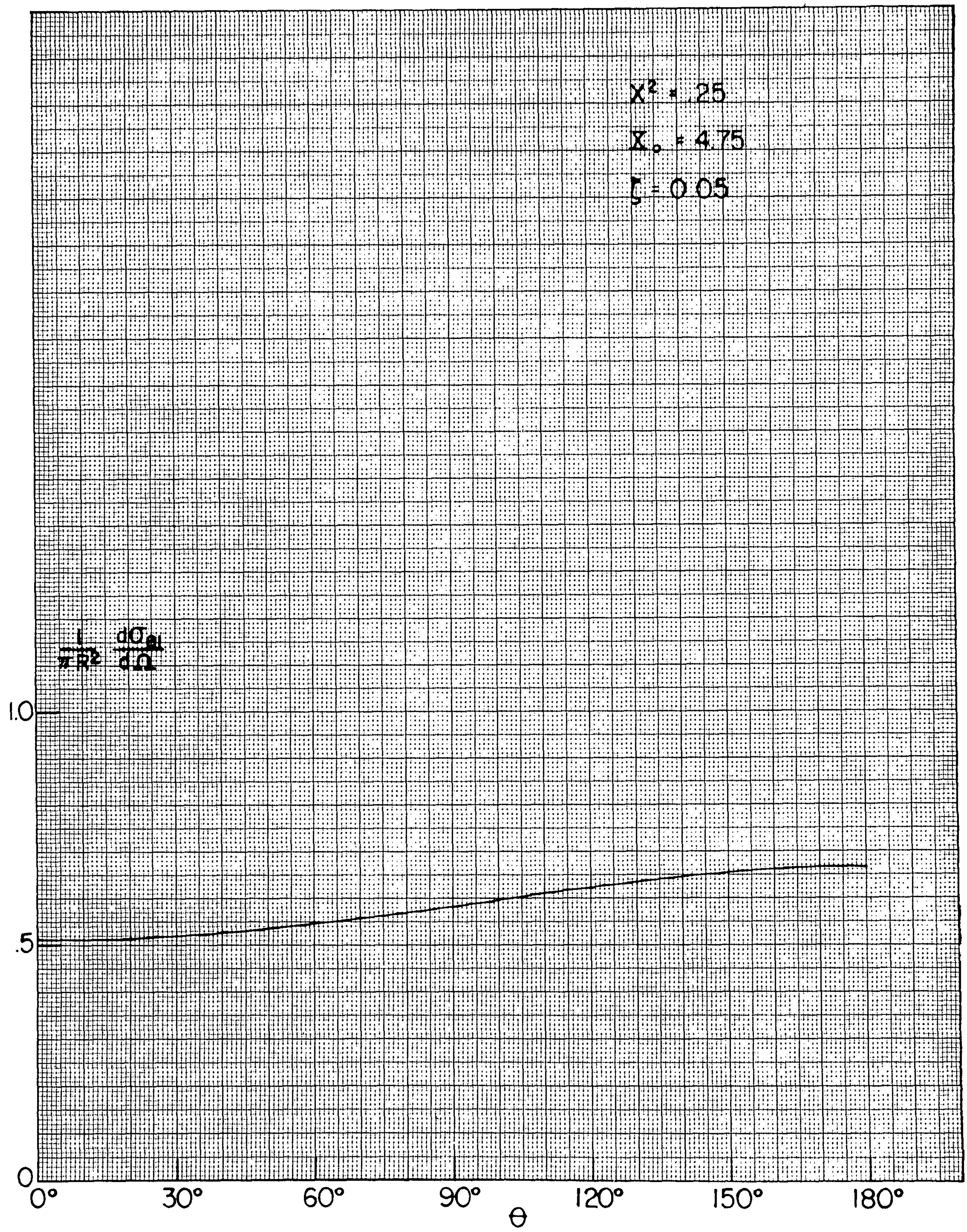


X = 2.6
ζ = 0.05
8
6
4
2
0
$\frac{\sigma_{tot}}{\pi R^2}$
0
20
40
60
80
X_o^2

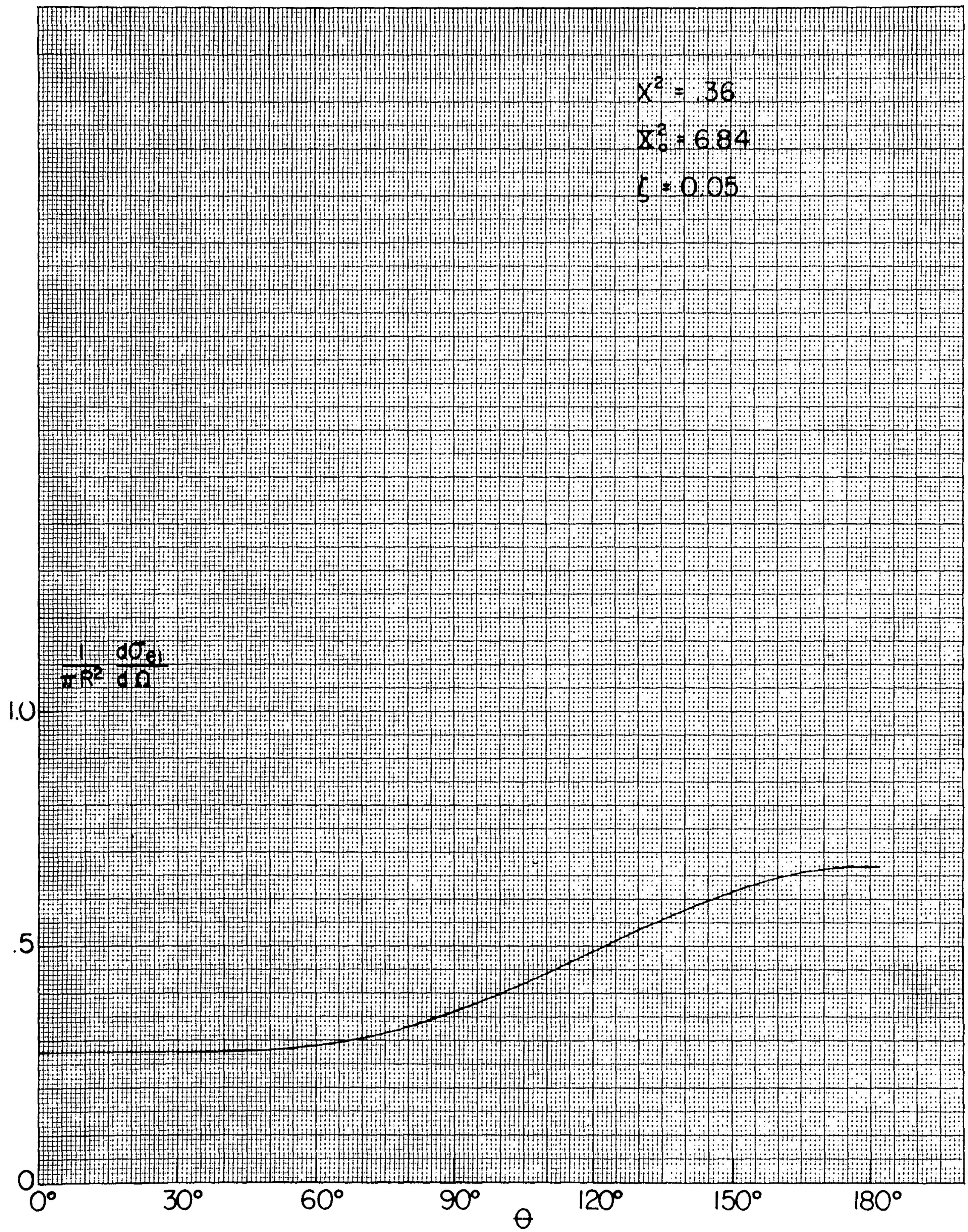

X = 2.8
ζ = 0.05
σ tot / π R²
0
2
4
6
8
0
20
40
60
80
X₀²

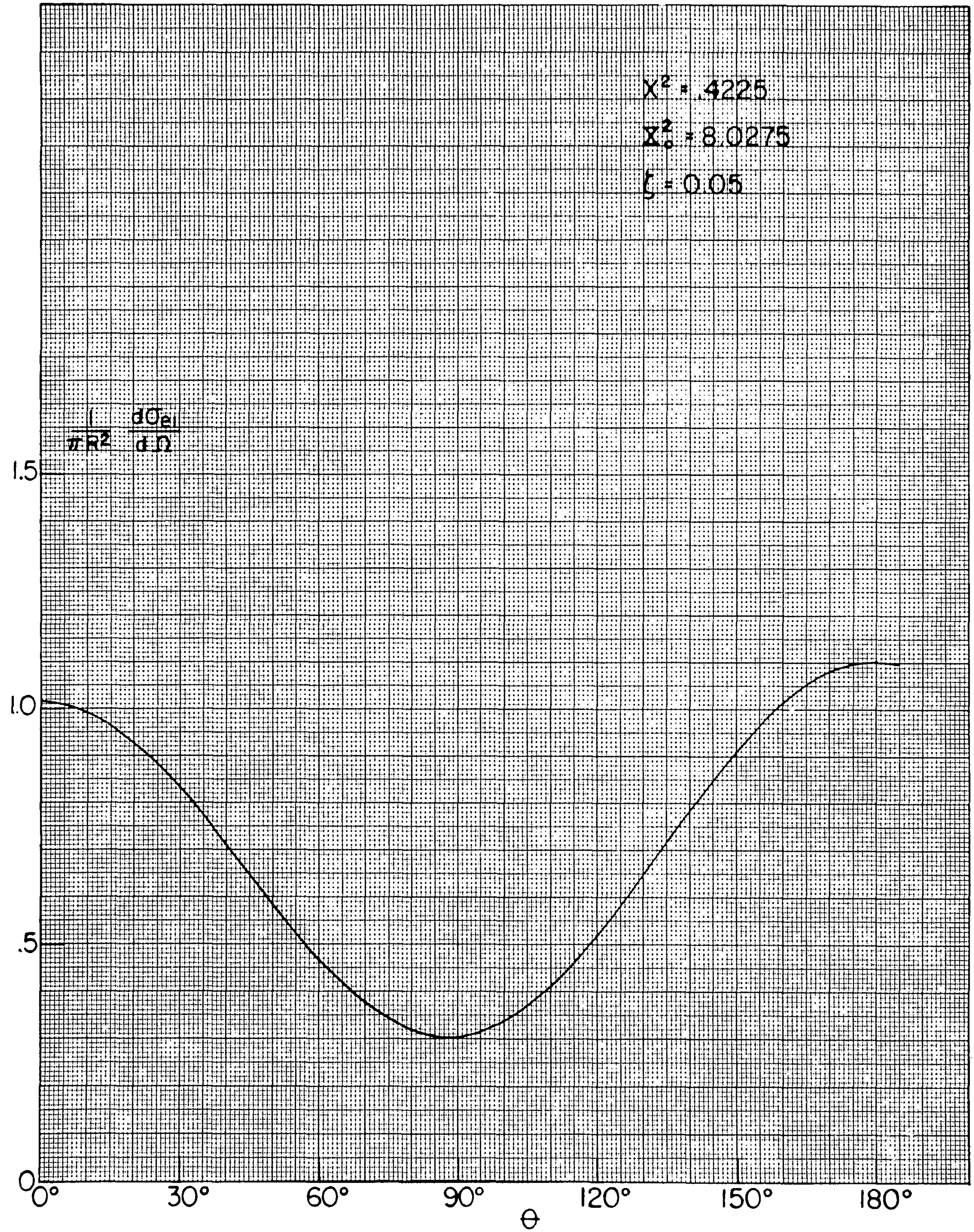

X = 3.0
ζ = 0.05
$\frac{\sigma_{tot}}{\pi R^2}$
0
2
4
6
0
20
40
60
80
X_o^2

IV. PLOTS OF DIFFERENTIAL CROSS SECTION FOR ELASTIC SCATTERING (SHAPE-ELASTIC ONLY) FOR $x^2/X_o^2 = 1/19$ WITH $\zeta = 0.05$

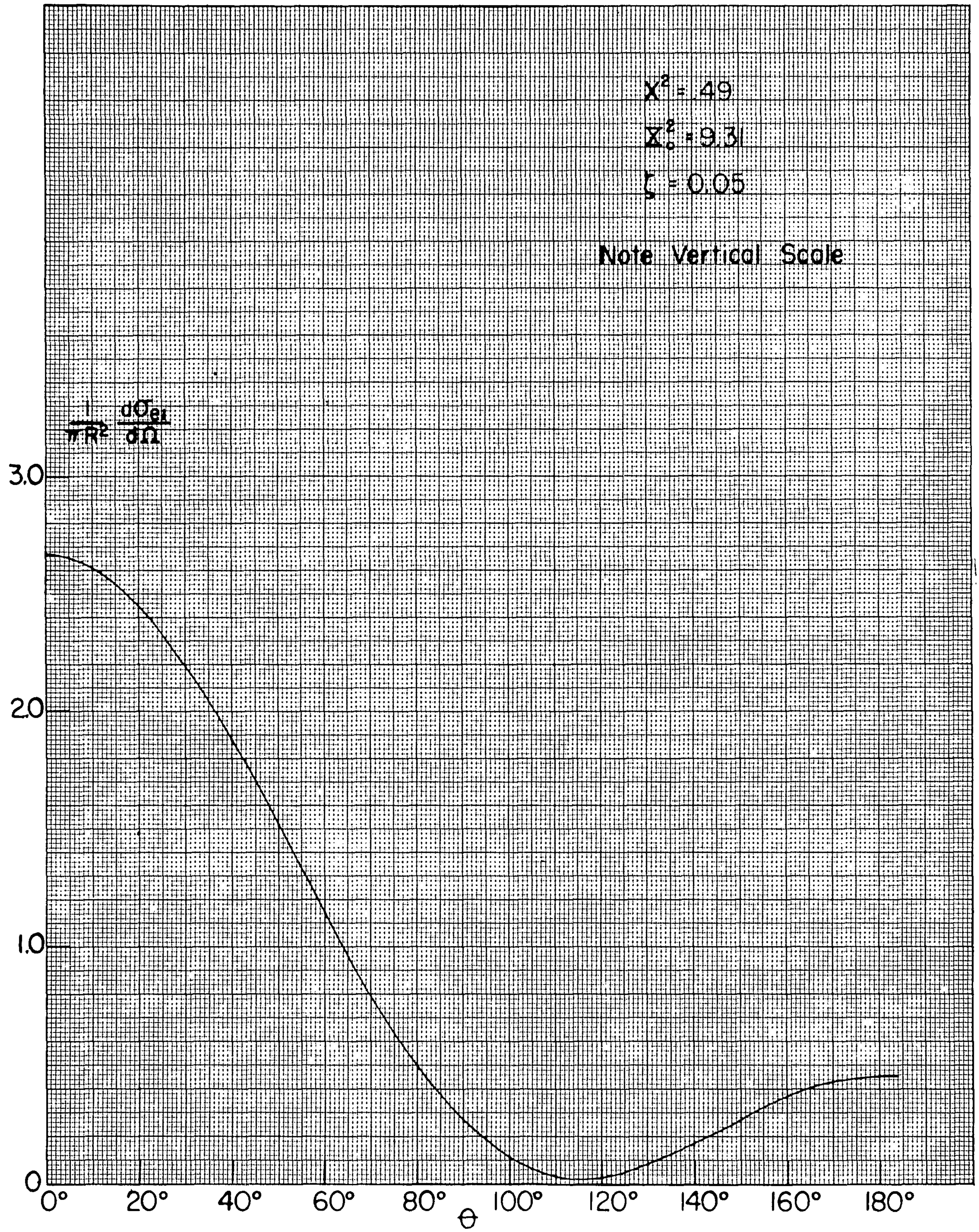


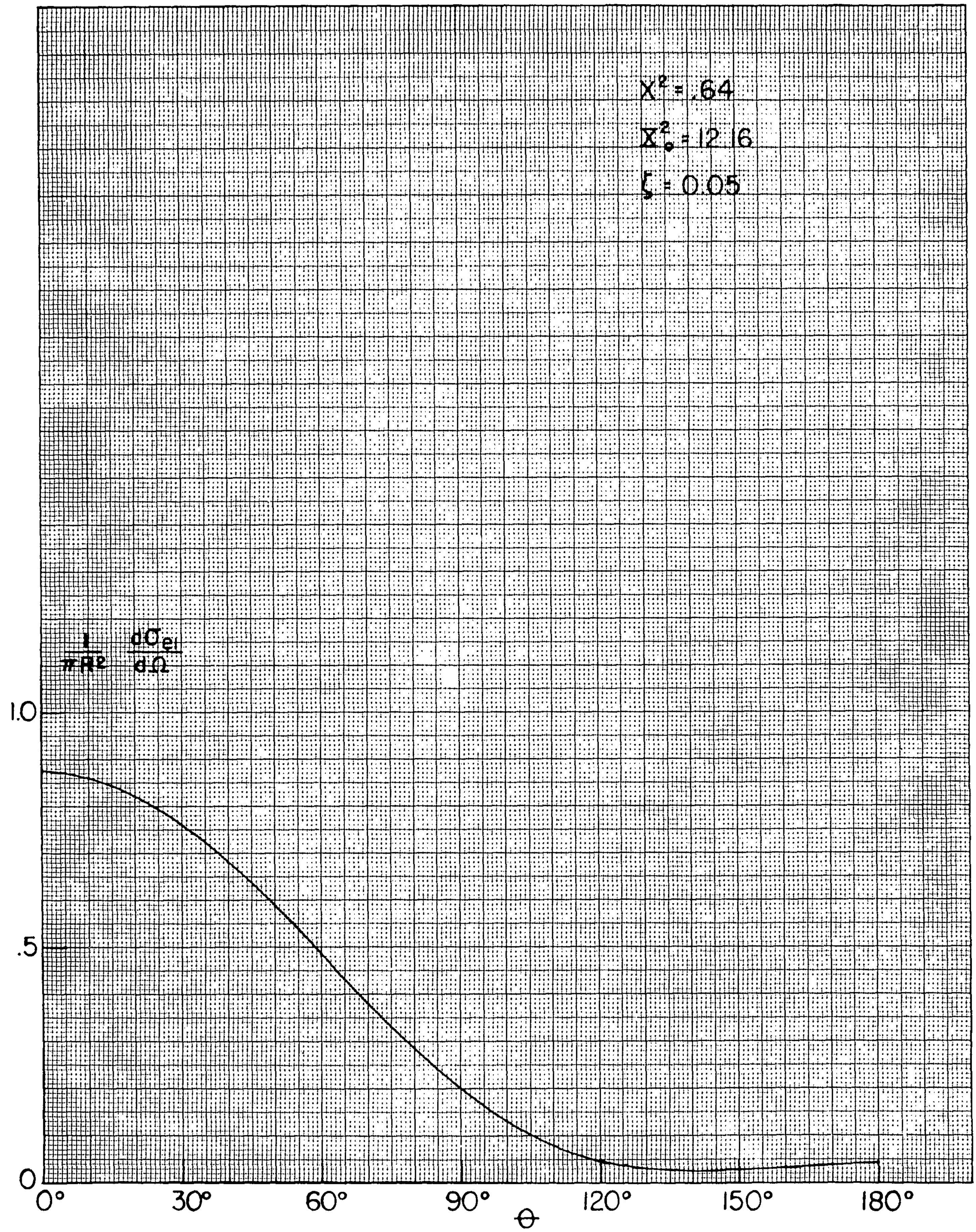
X² = .25
X₀ = 4.75
ζ = 0.05
$\frac{1}{\pi R^2}\frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ


$\chi^2 = .36$
$\chi_0^2 = 6.84$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

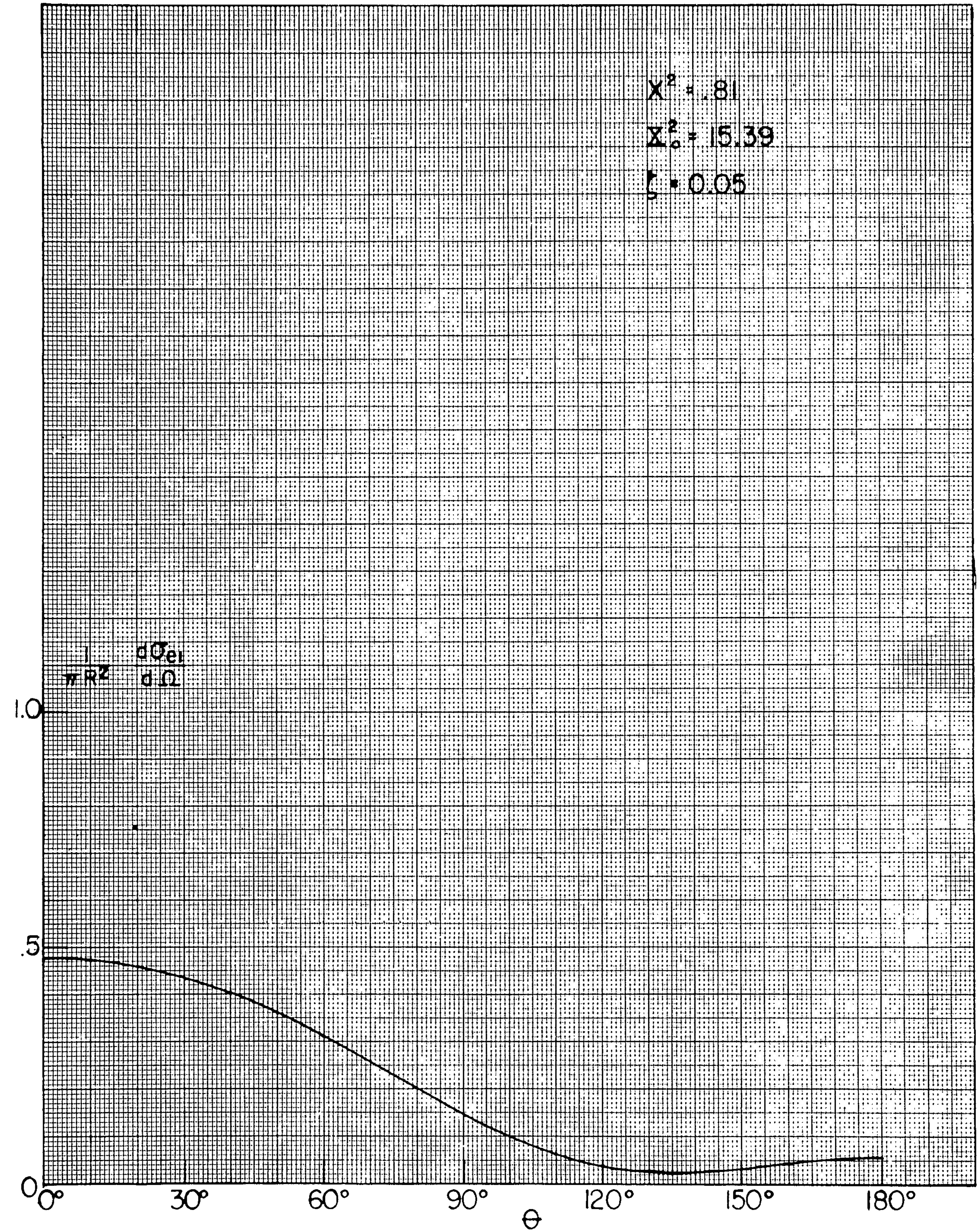


$X^2 = .4225$
$X_0^2 = 8.0275$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

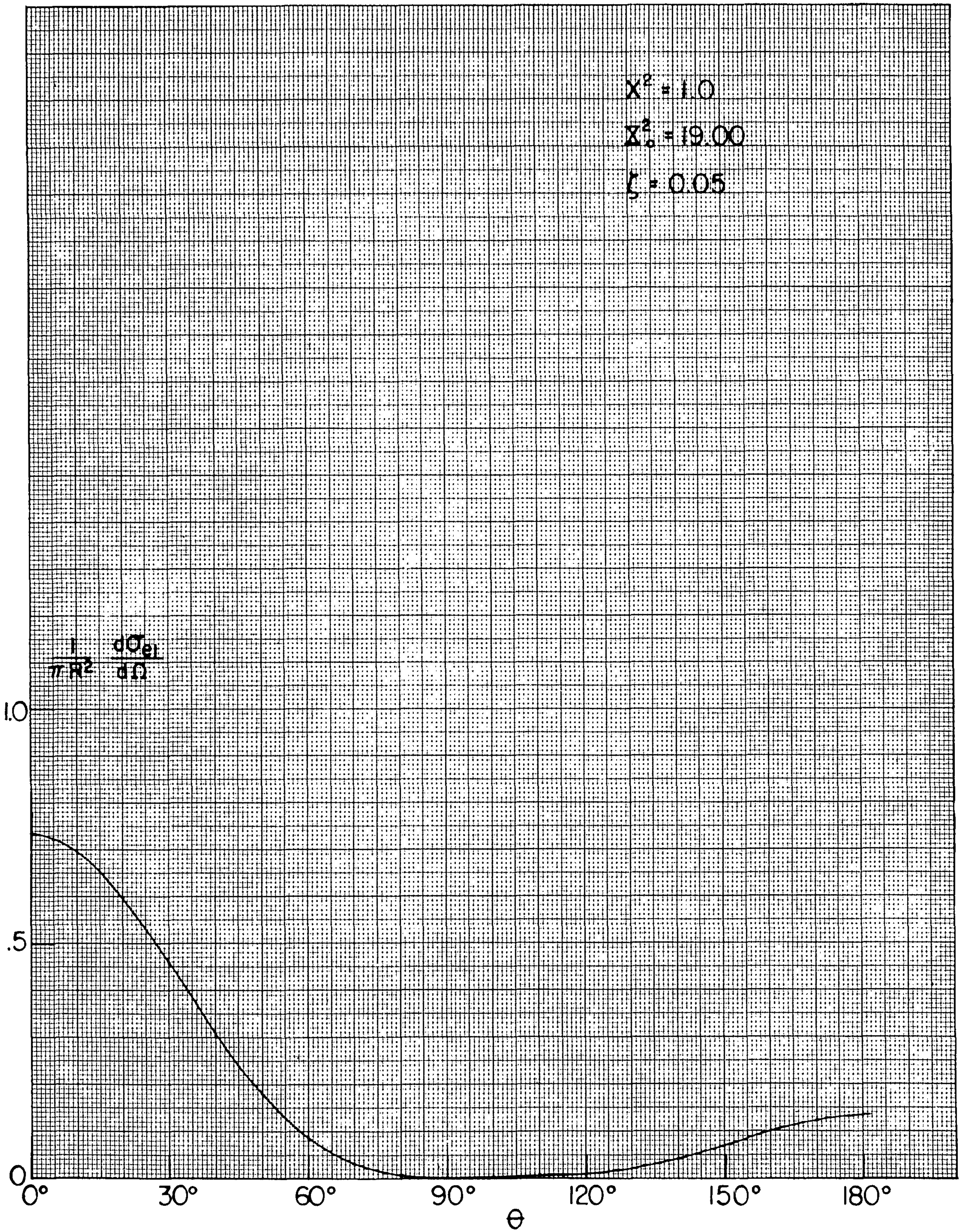

$X^2 = .49$
$X_0^2 = 9.31$
$\zeta = 0.05$
Note Vertical Scale
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
3.0
2.0
1.0
0
0°
20°
40°
60°
80°
100°
120°
140°
160°
180°
θ

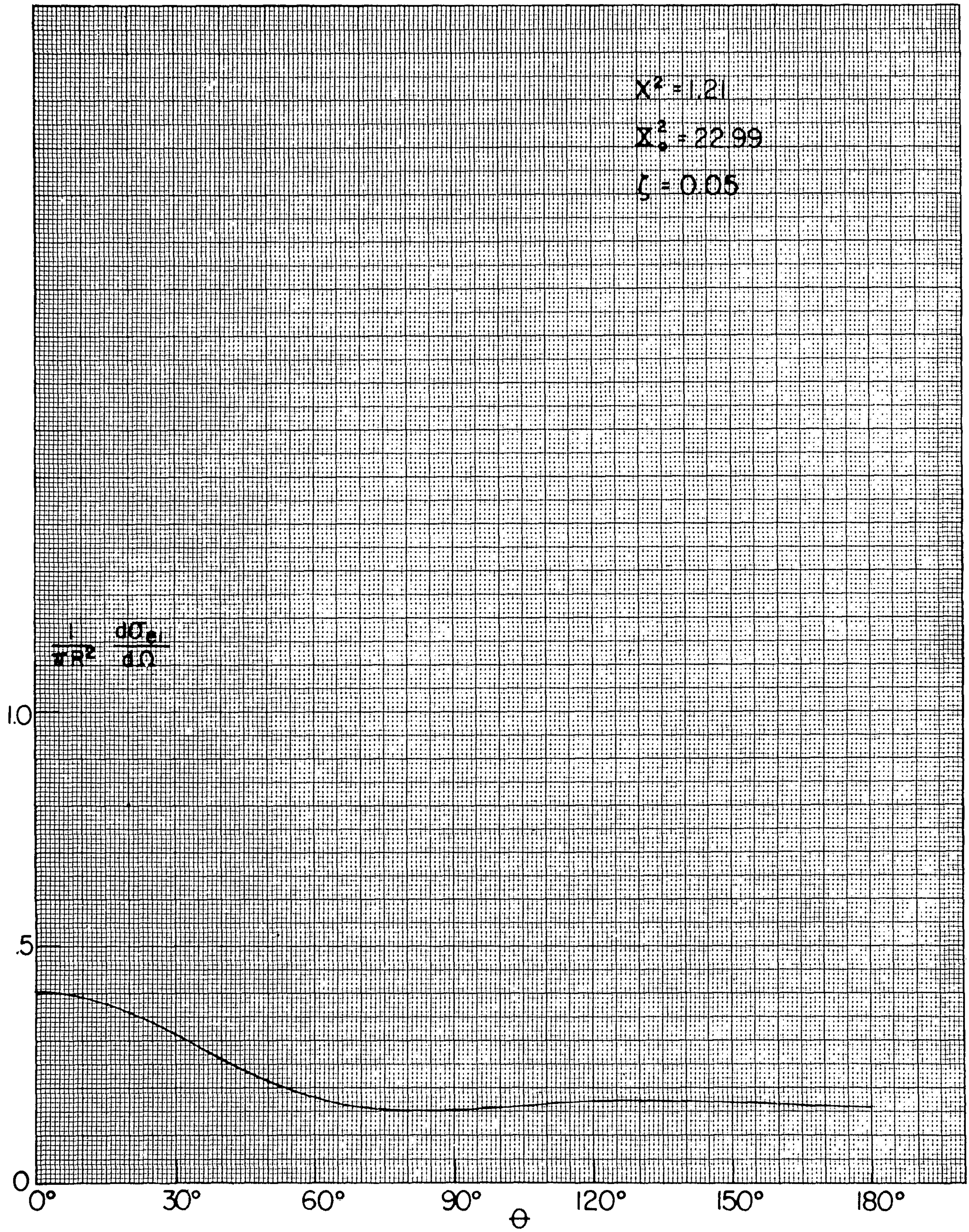


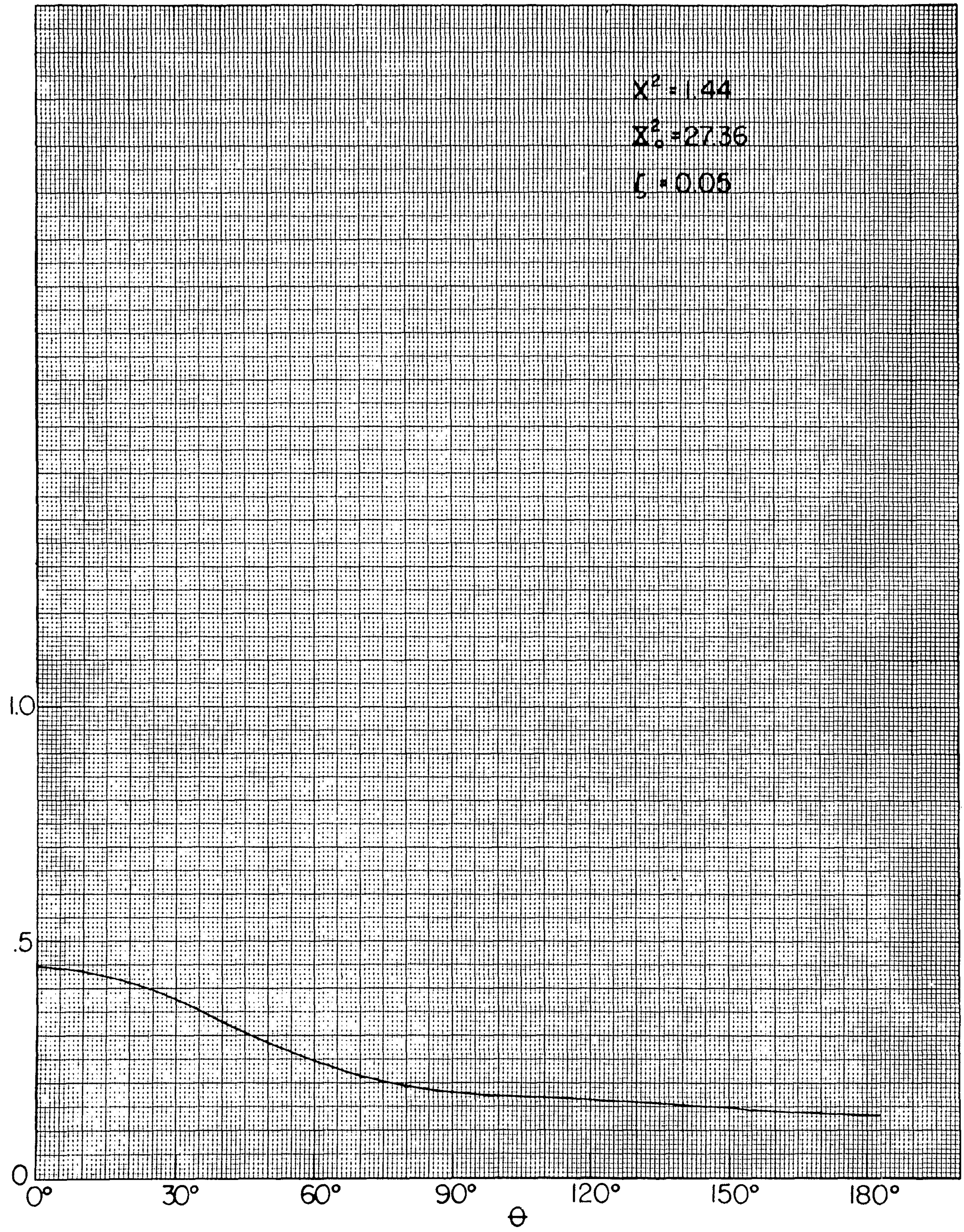
$X^2 = .64$
$X_0^2 = 12.16$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



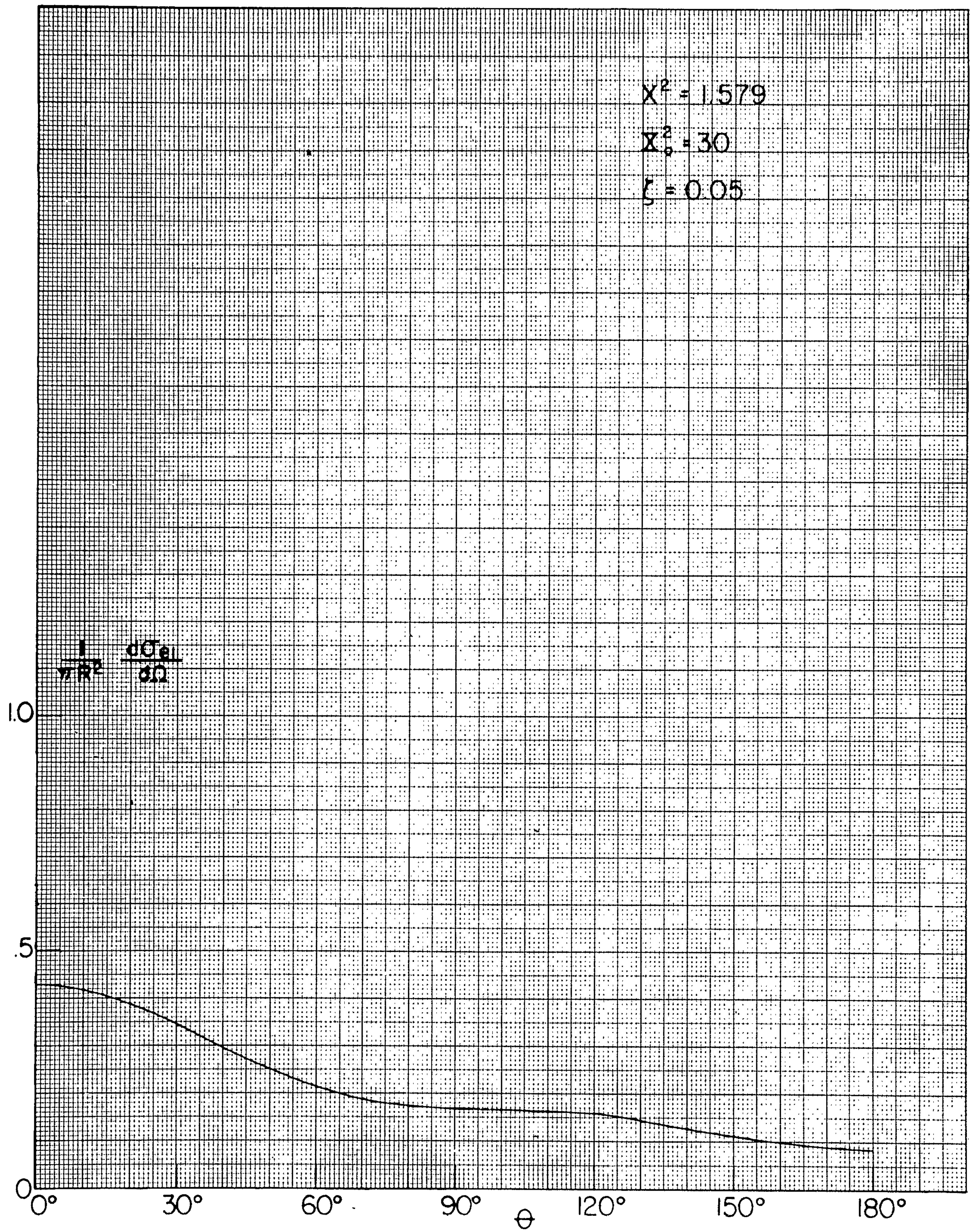
$X^2 = .81$
$X_o^2 = 15.39$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

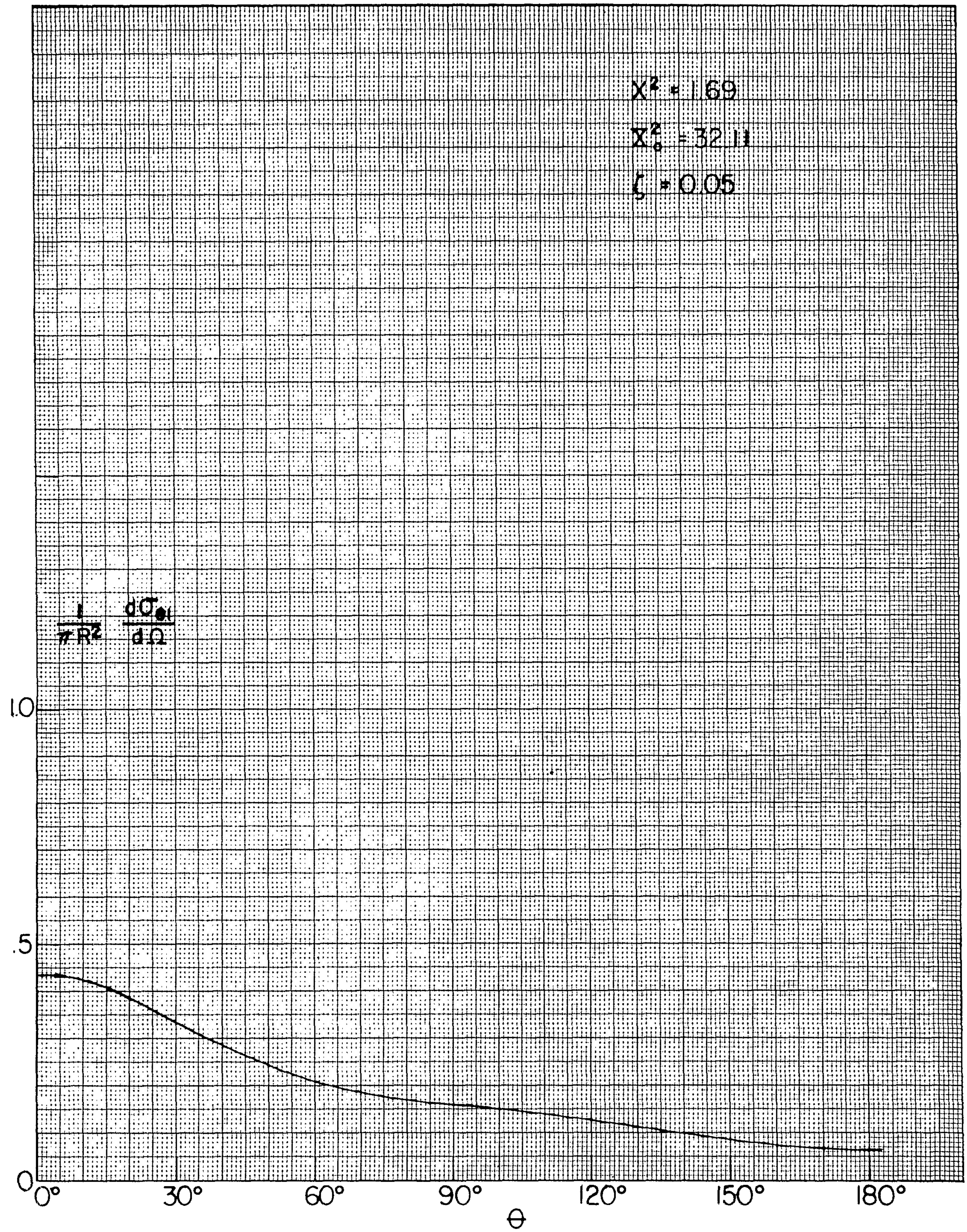

$X^2 = 1.0$
$\bar{X}_0^2 = 19.00$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ


$X^2 = 1.21$
$X_0^2 = 22.99$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

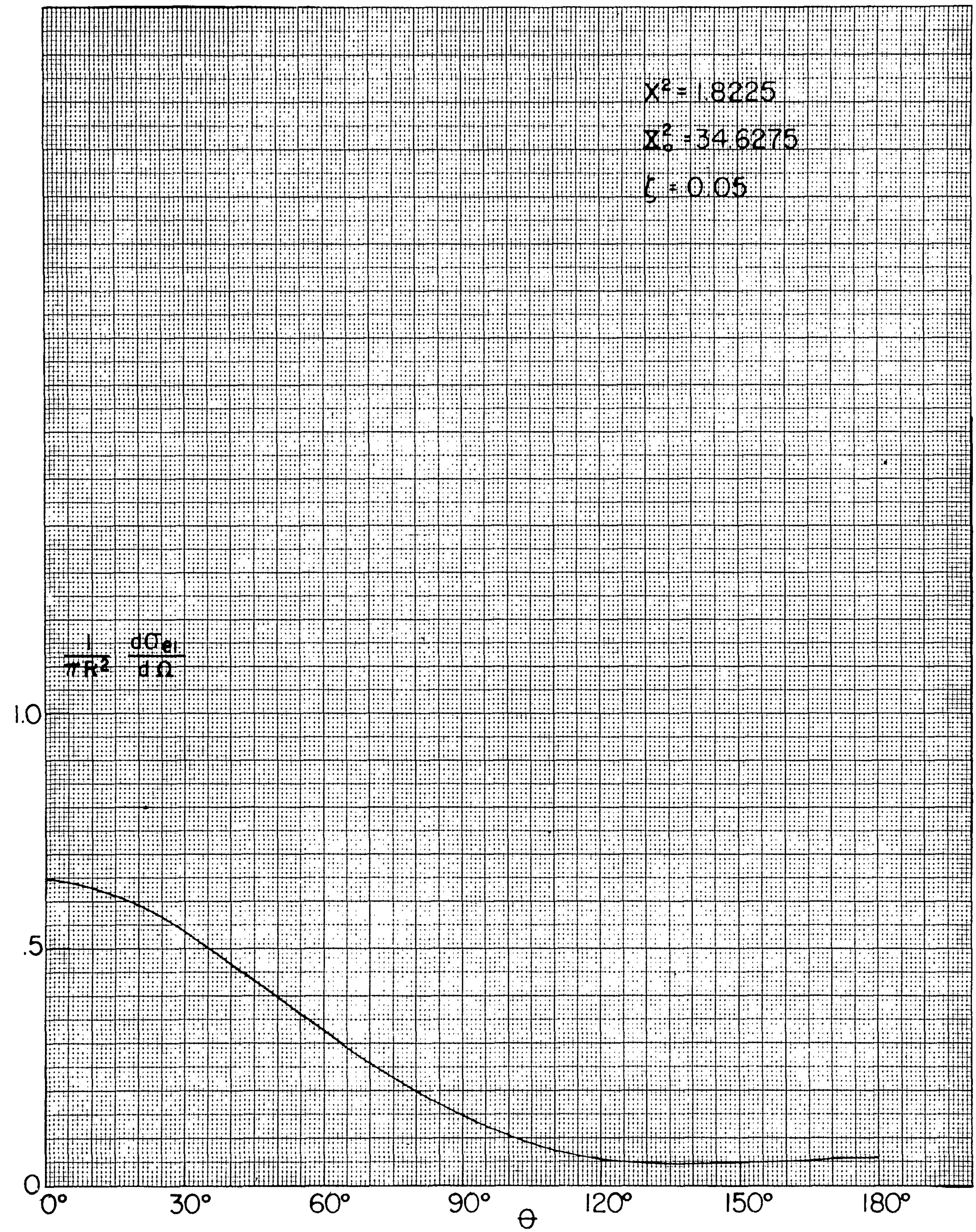


$X^2 = 1.44$
$X_0^2 = 27.36$
$\zeta = 0.05$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

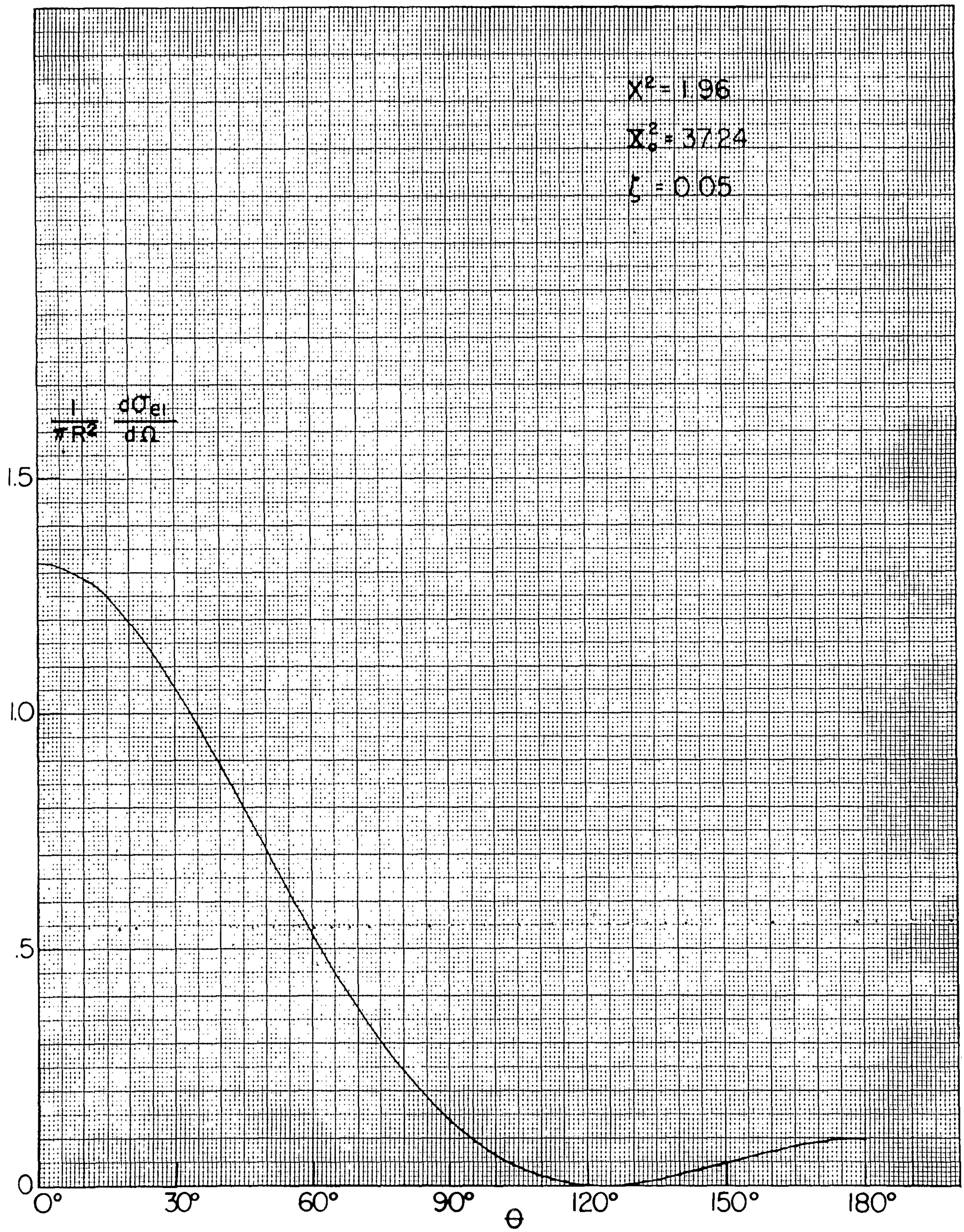

$X^2 = 1.579$
$X_o^2 = 30$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



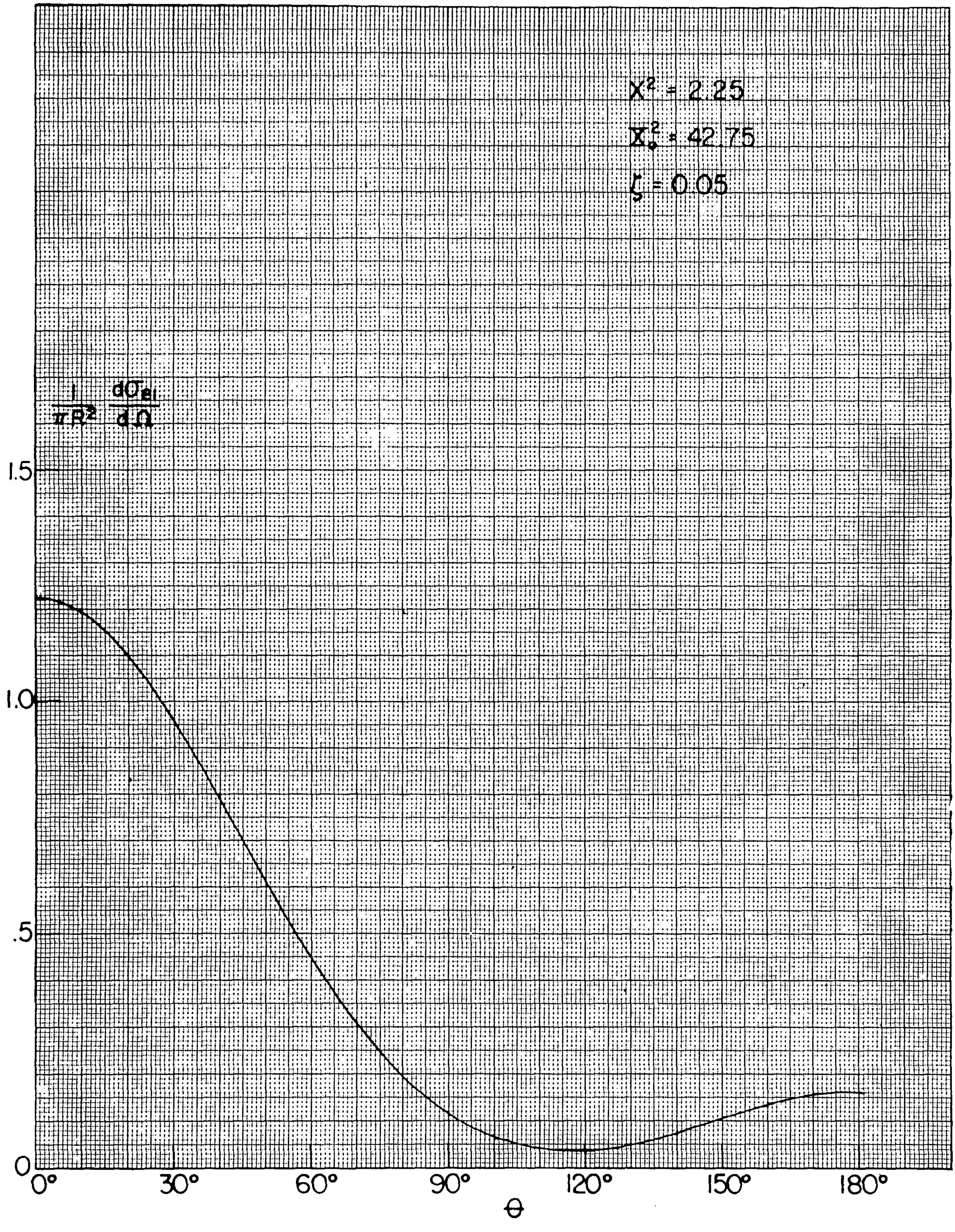
X² = 1.69
X²₀ = 32.11
ζ = 0.05
1/πR² dσ_el/dΩ
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



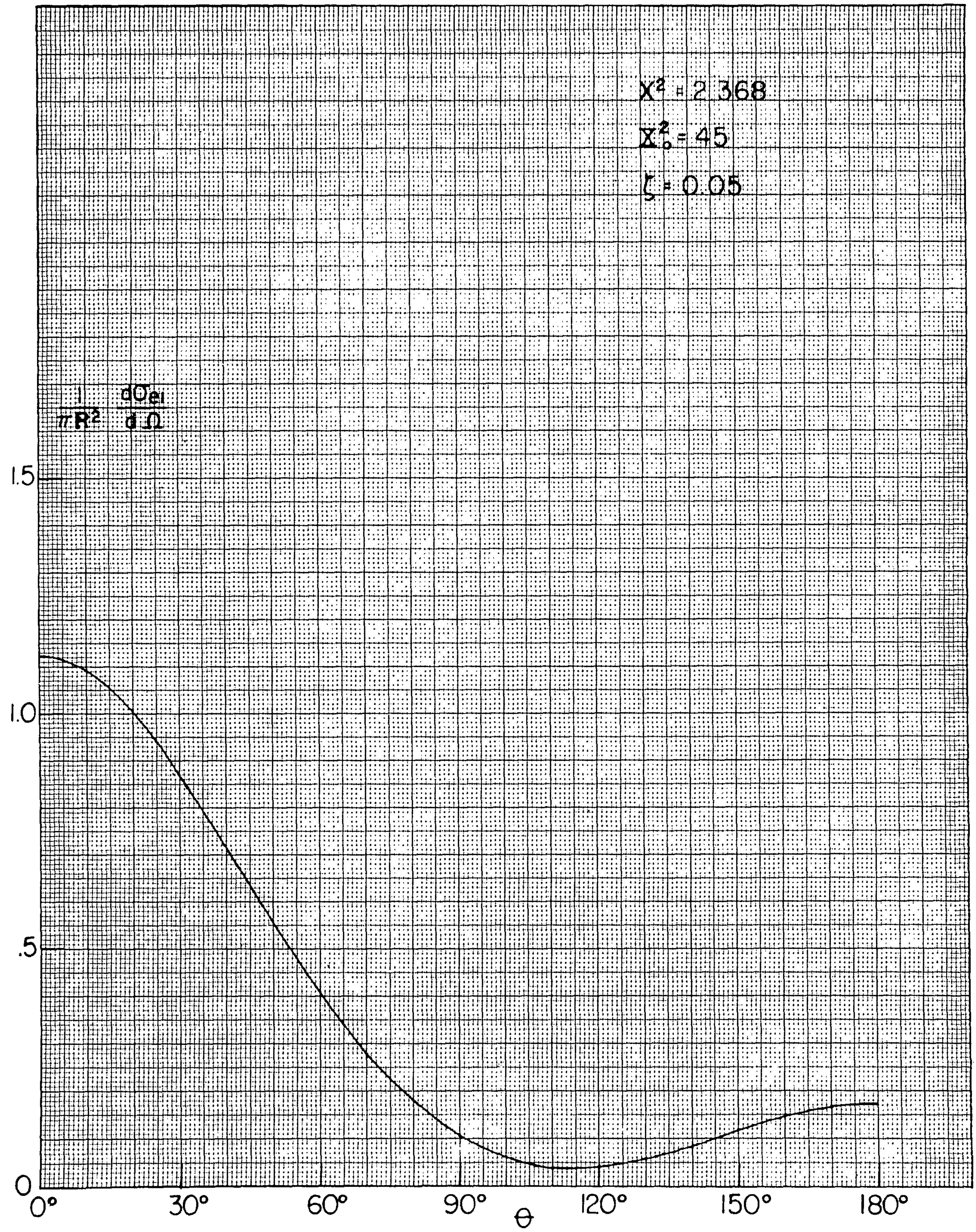
$X^2 = 1.8225$
$X_0^2 = 34.6275$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



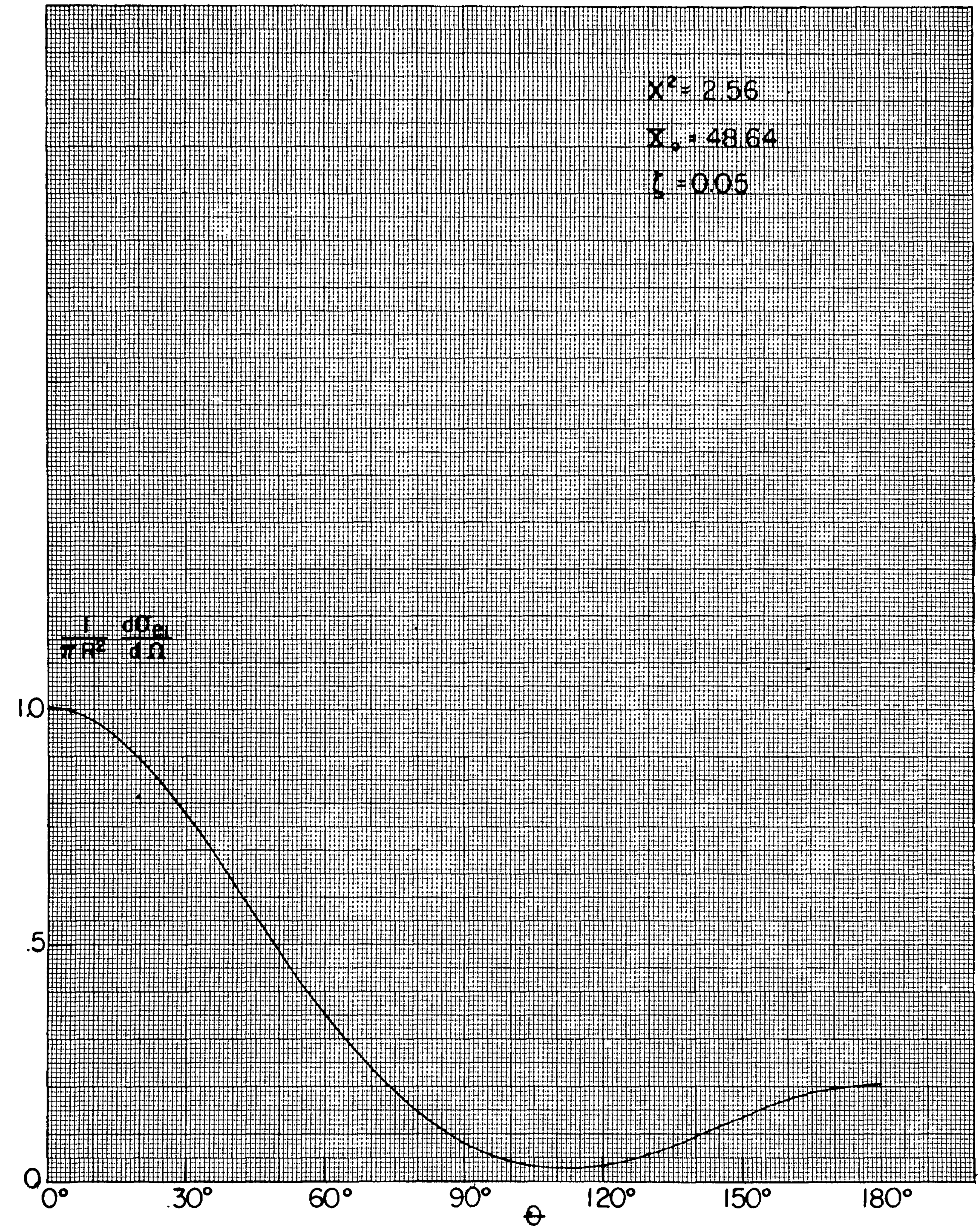
$X^2 = 1.96$
$X_0^2 = 37.24$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

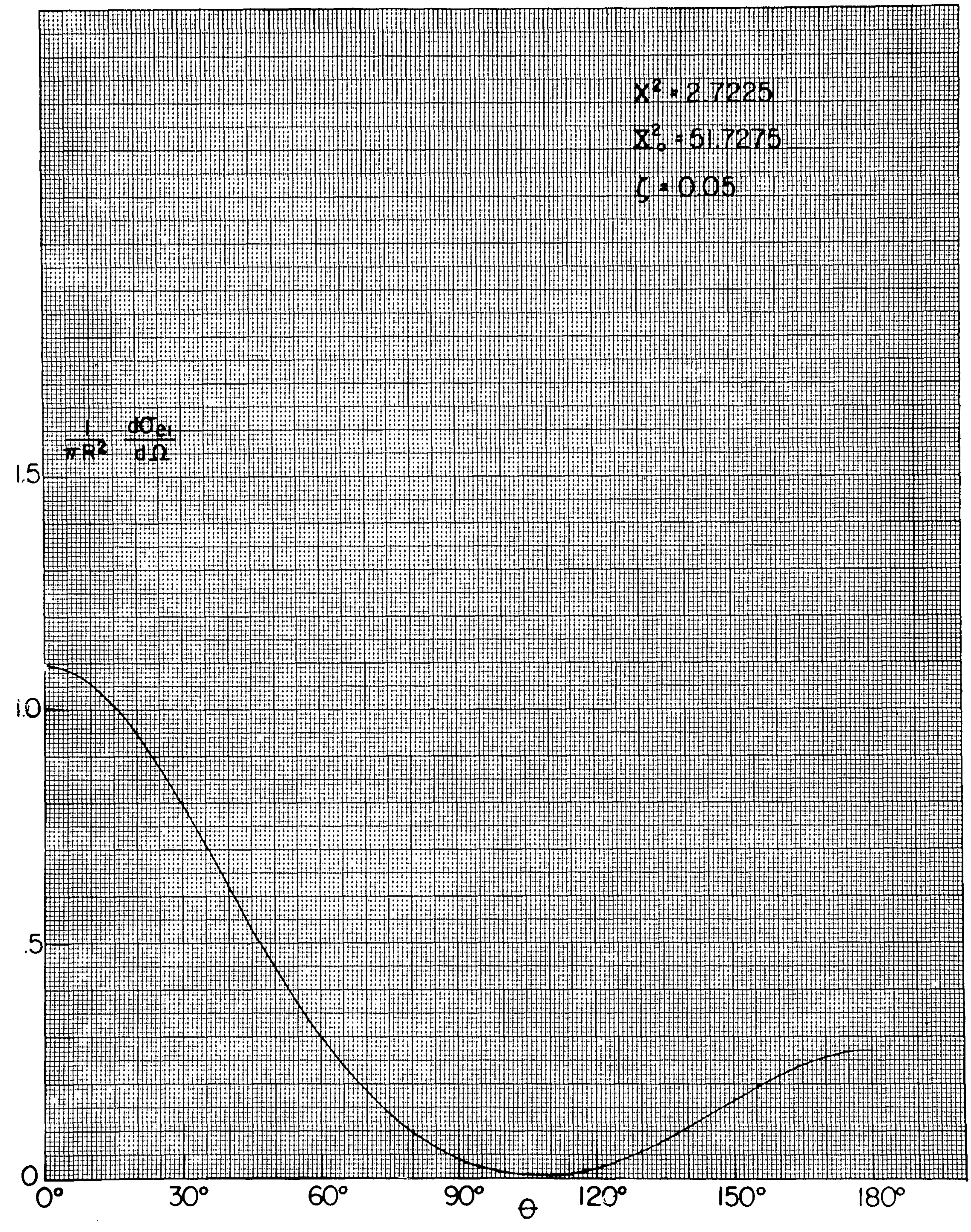


$X^2 = 2.25$
$X_0^2 = 42.75$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

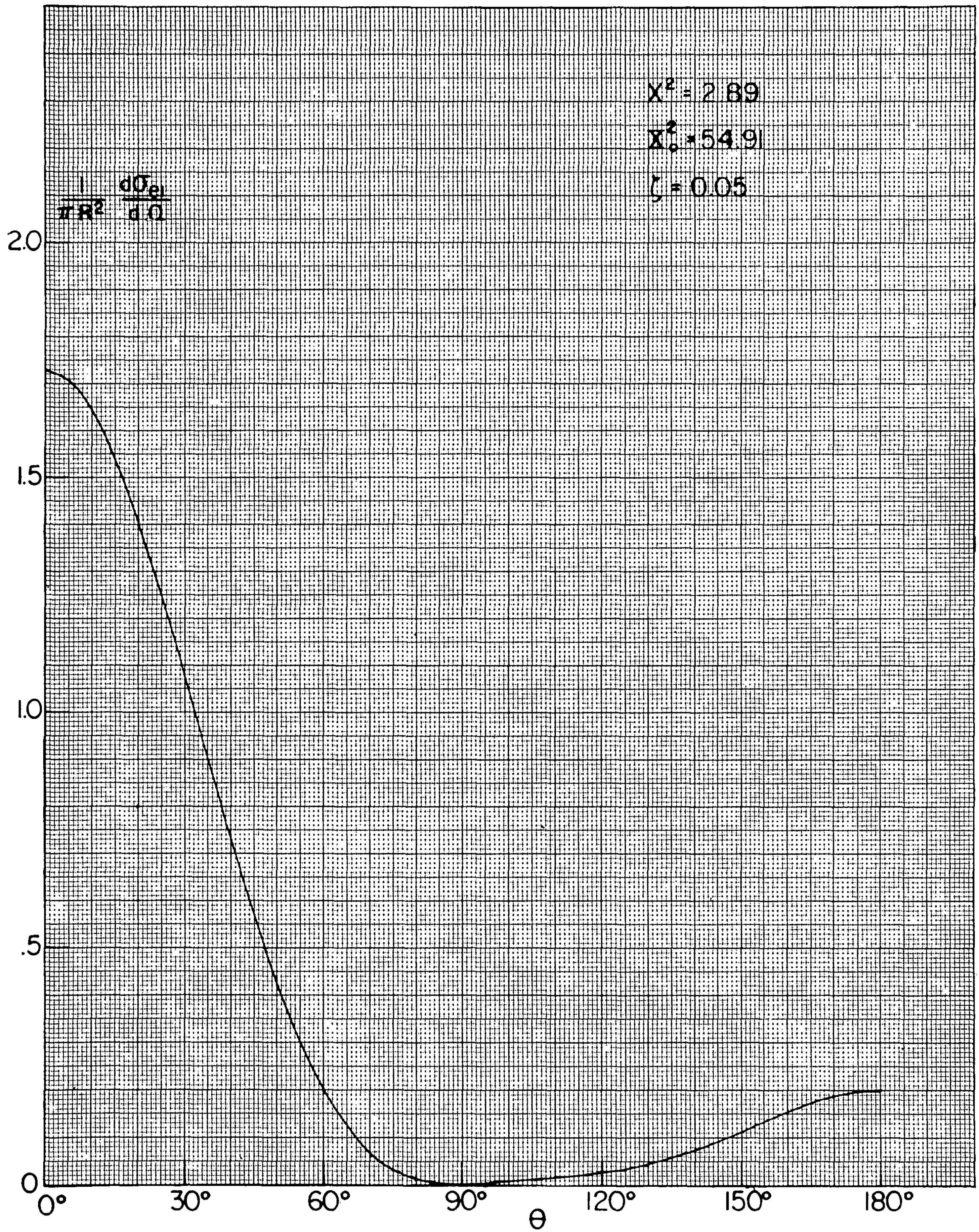


$X^2 = 2.368$
$X_0^2 = 45$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

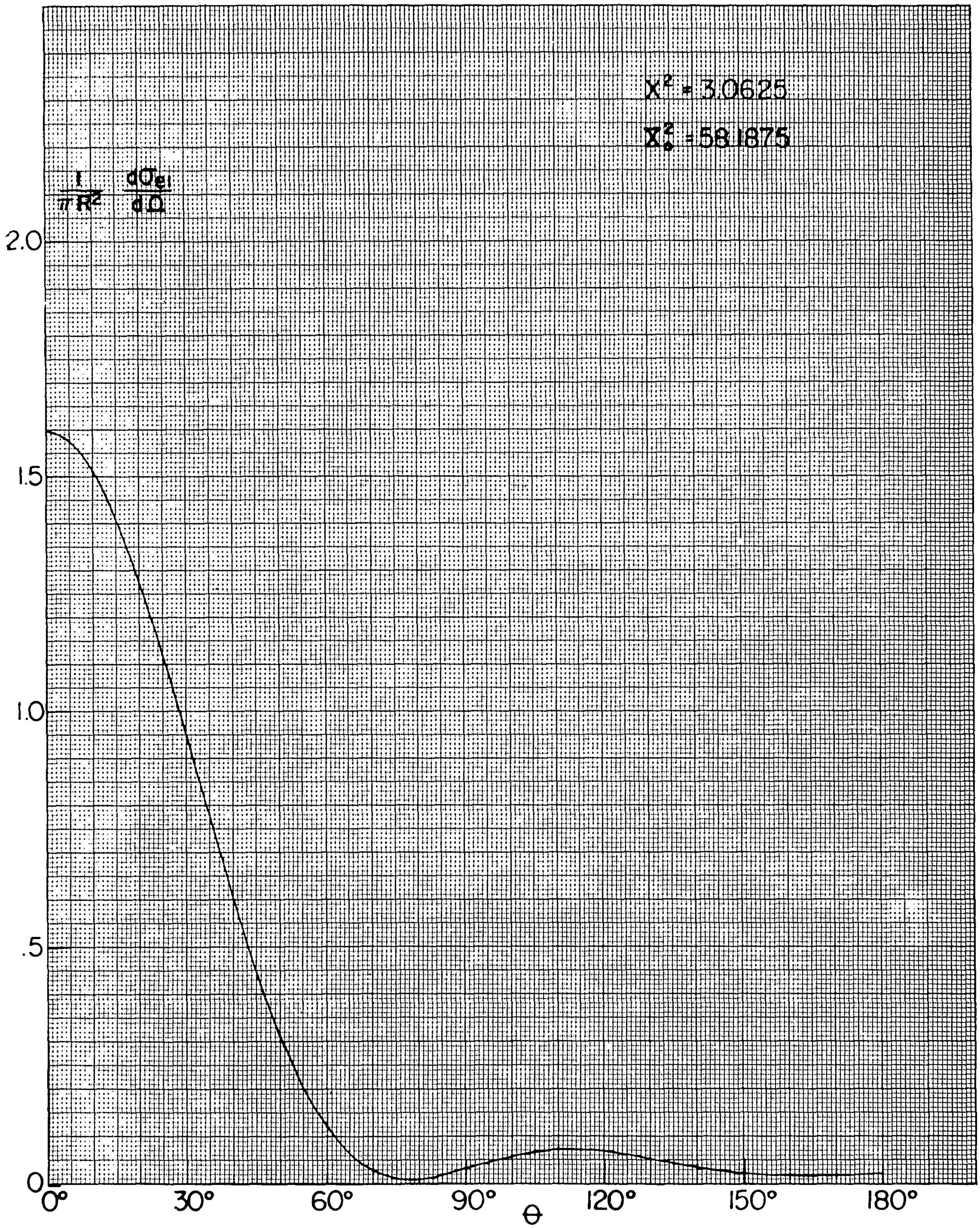

$X^2 = 2.56$
$X_0 = 48.64$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

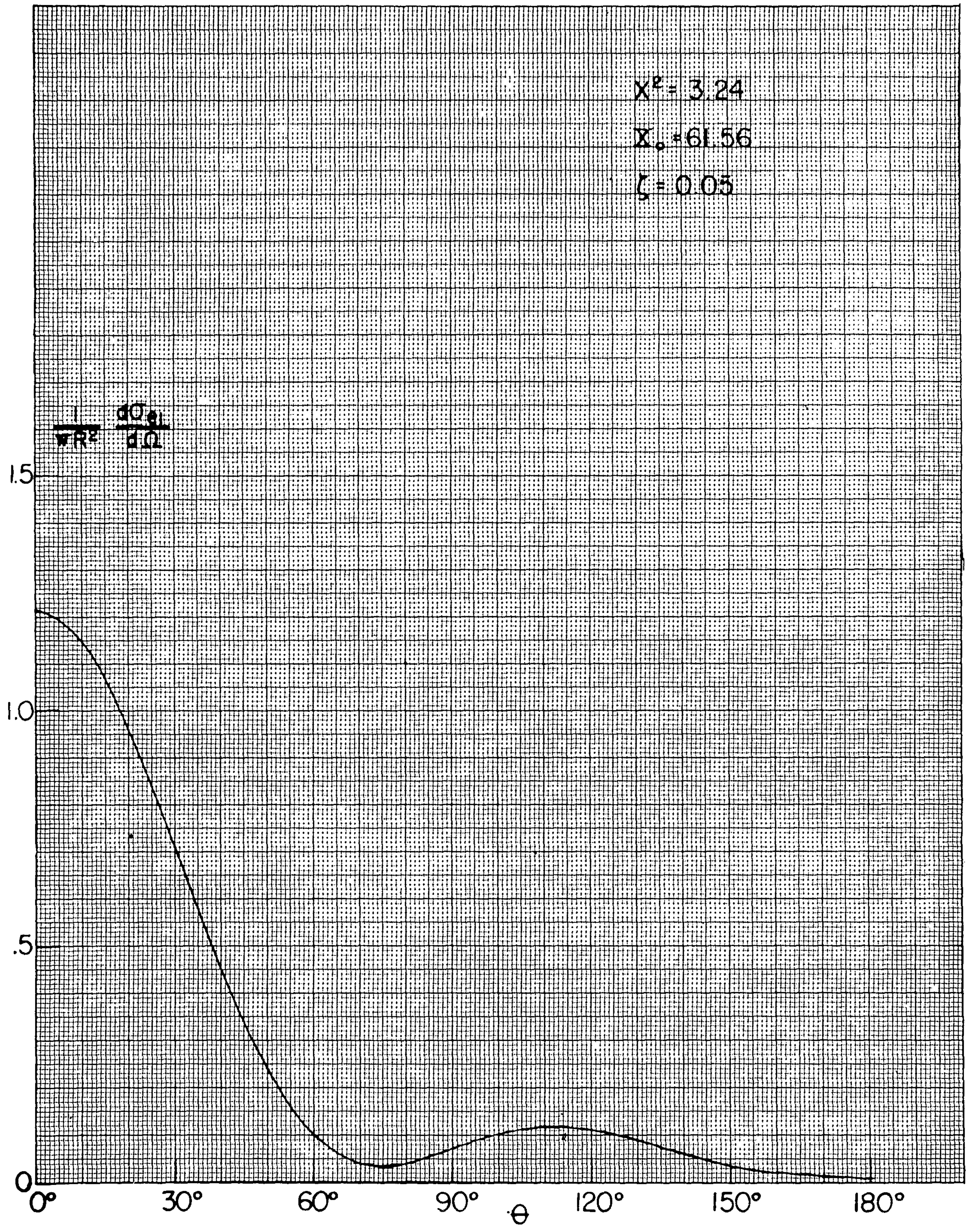


$\chi^2 = 2.7225$
$\chi^2_0 = 51.7275$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.5
1.0
.5
0
0°
30°
60°
90°
θ
120°
150°
180°

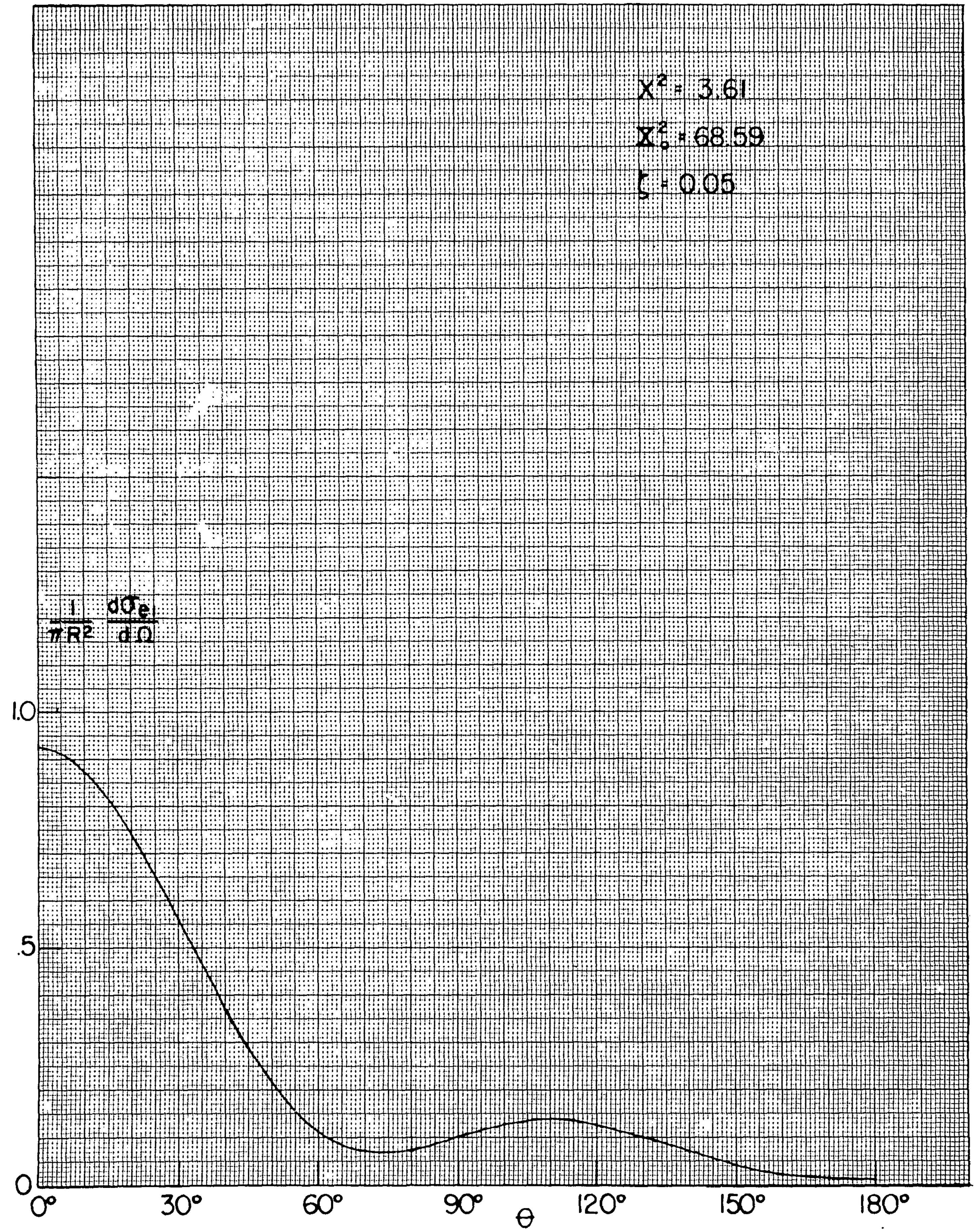


$X^2 = 2.89$
$X_o^2 = 54.91$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
2.0
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

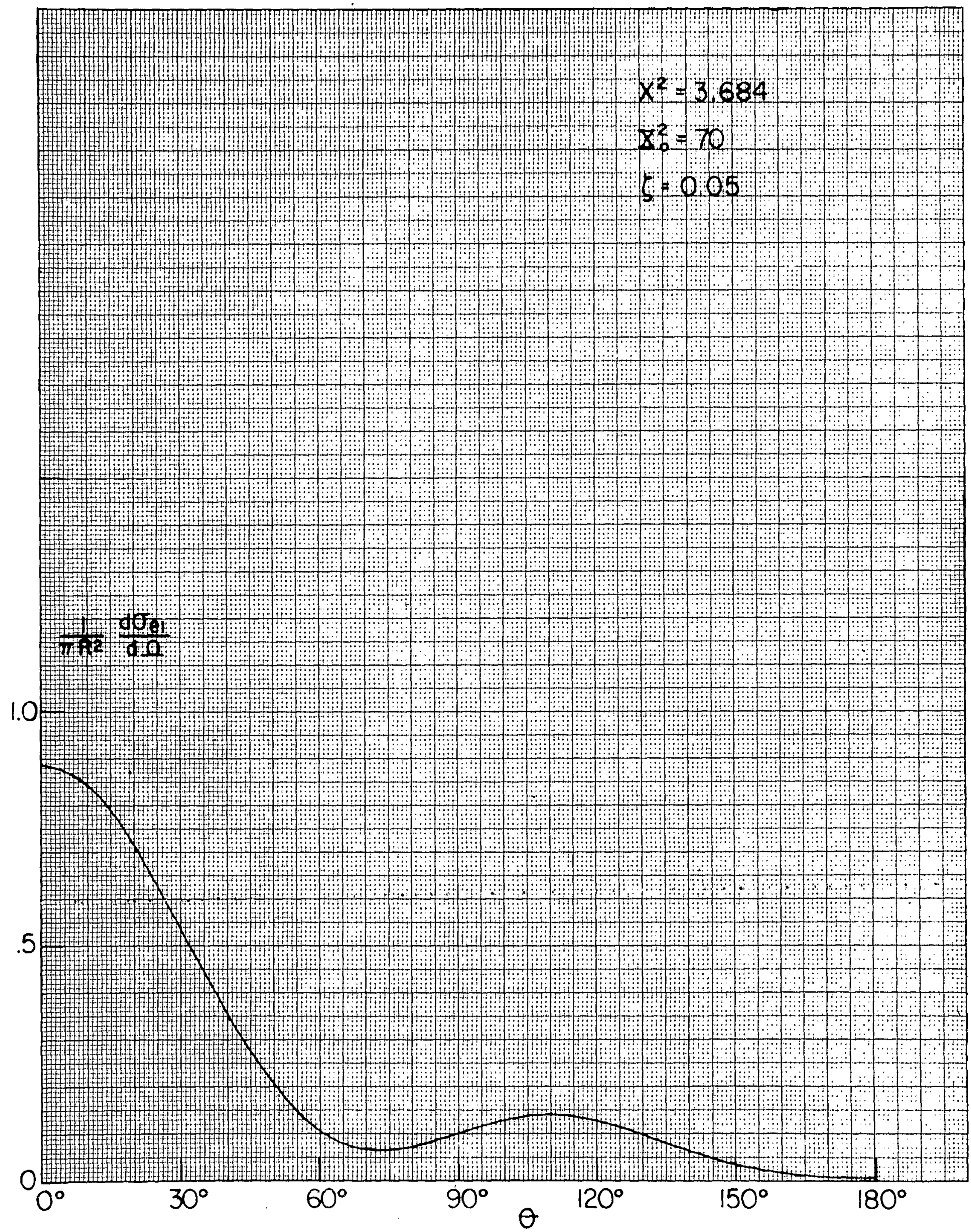

$X^2 = 3.0625$
$X_o^2 = 58.1875$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
2.0
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



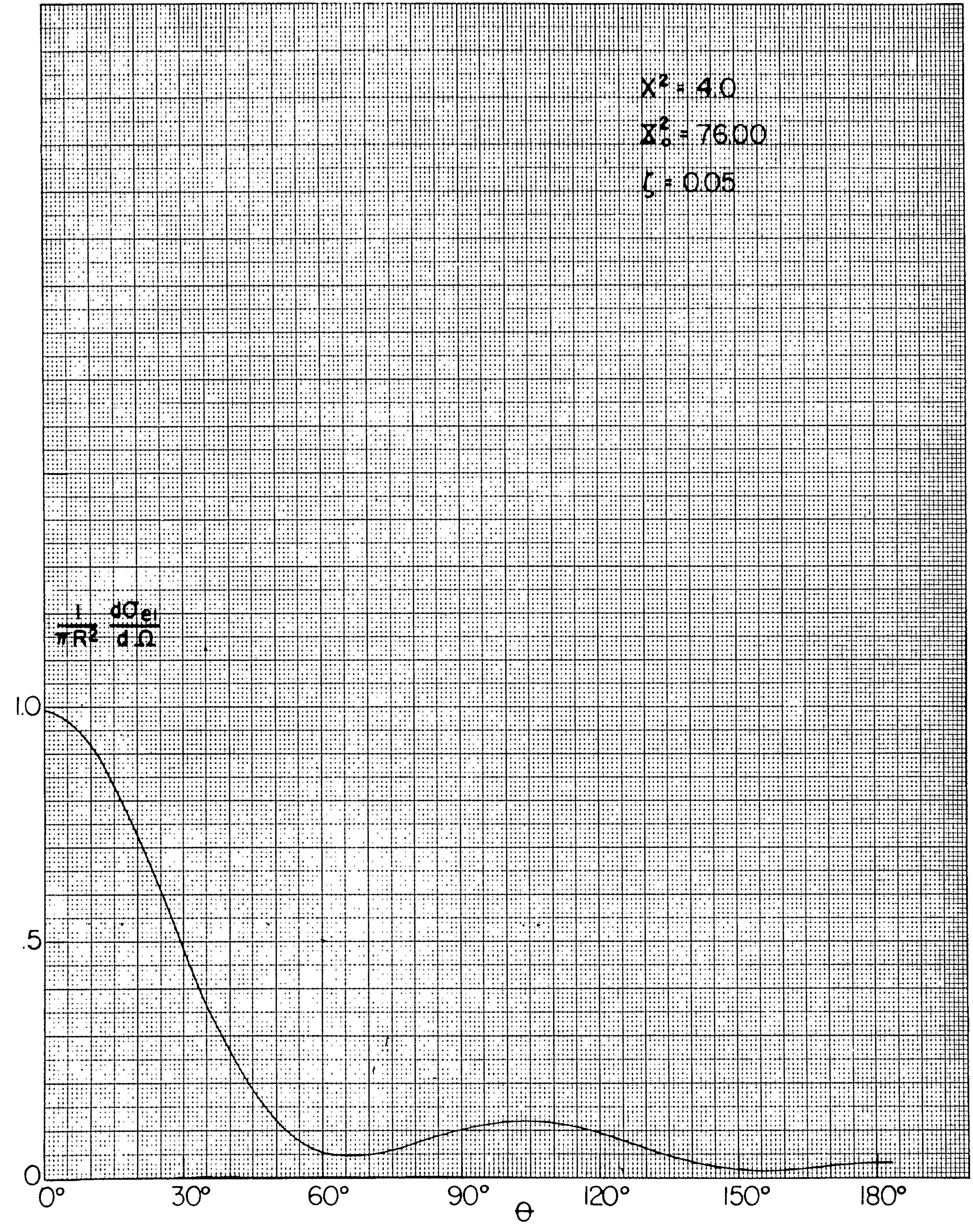
$X^2 = 3.24$
$X_0 = 61.56$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.5
1.0
.5
0
0°
30°
60°
90°
θ
120°
150°
180°

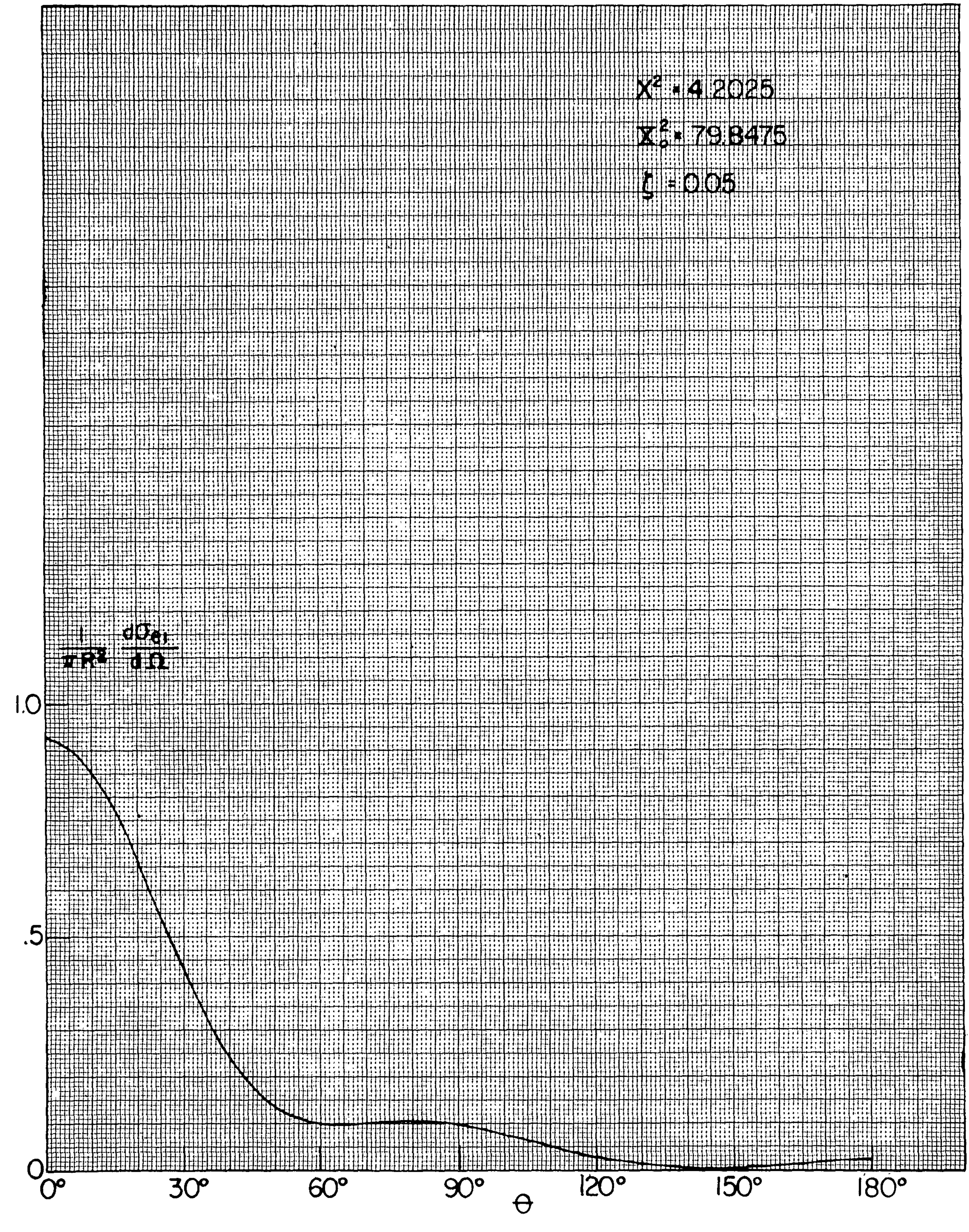


X² = 3.61
X₀² = 68.59
ζ = 0.05
1/πR² dσ_el/dΩ
1.0
.5
0
0°
30°
60°
90°
θ
120°
150°
180°

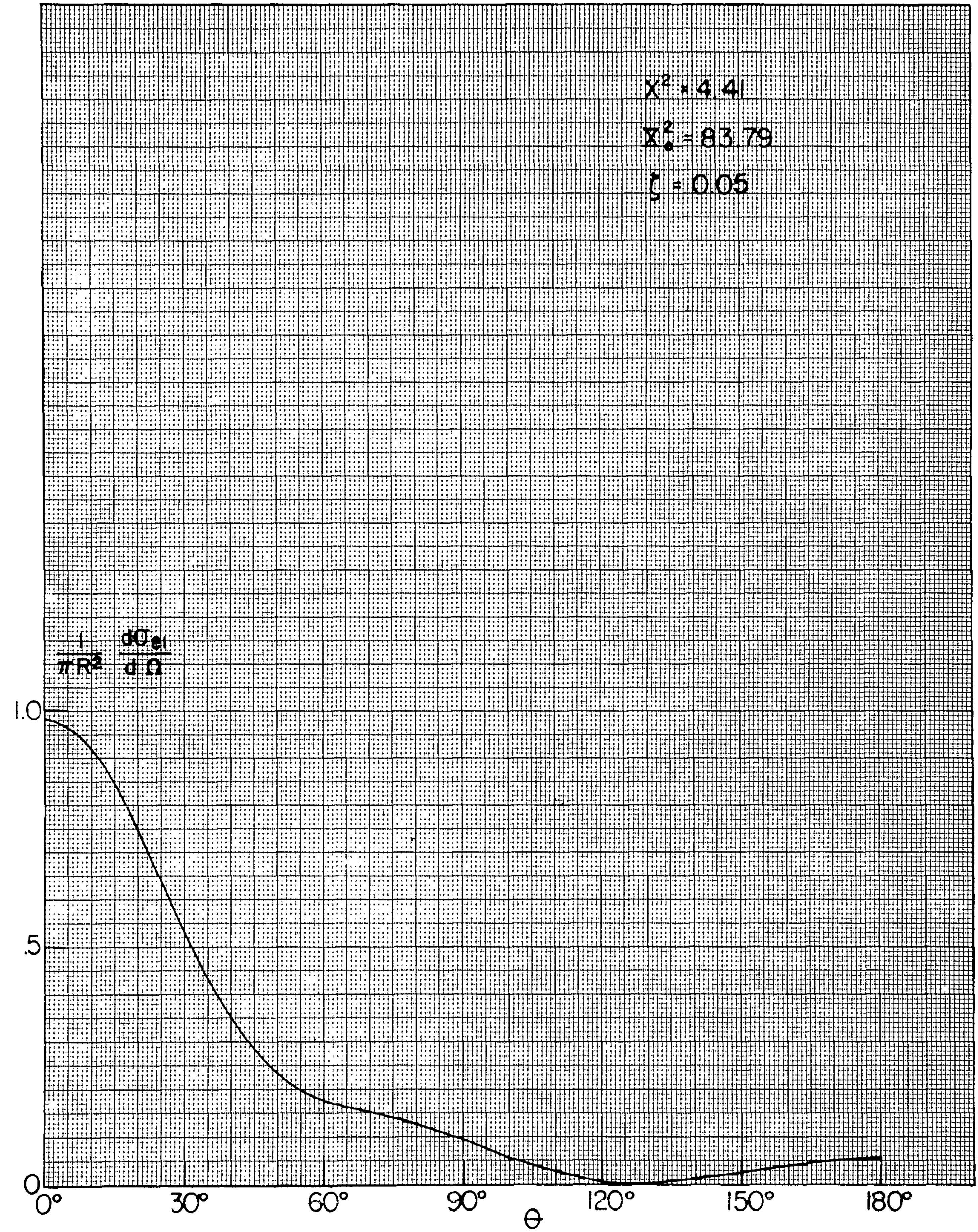


$X^2 = 3.684$
$X_0^2 = 70$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ


$X^2 = 4.0$
$\bar{X}_0^2 = 76.00$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

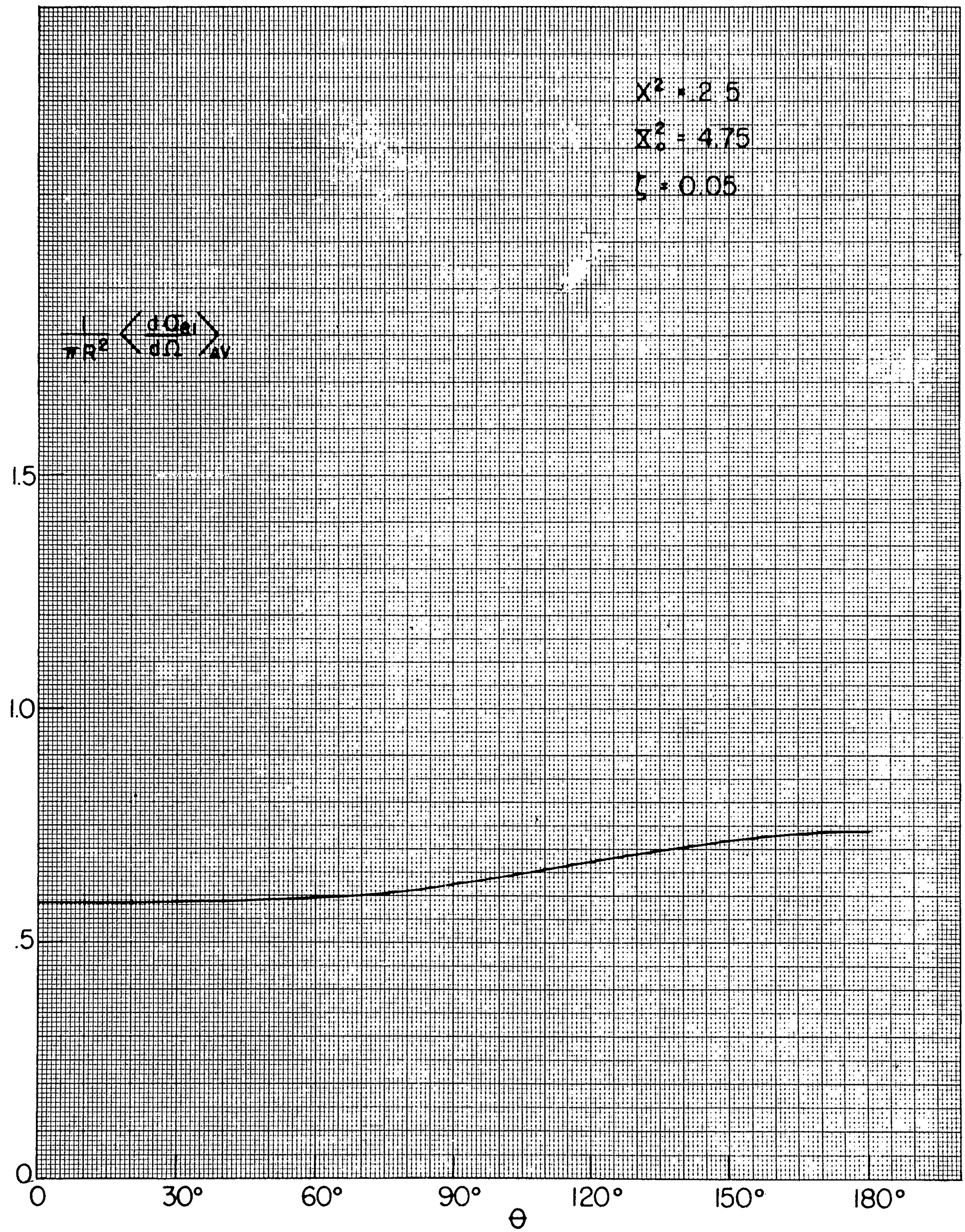


$X^2 = 4.2025$
$X_0^2 = 79.8475$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

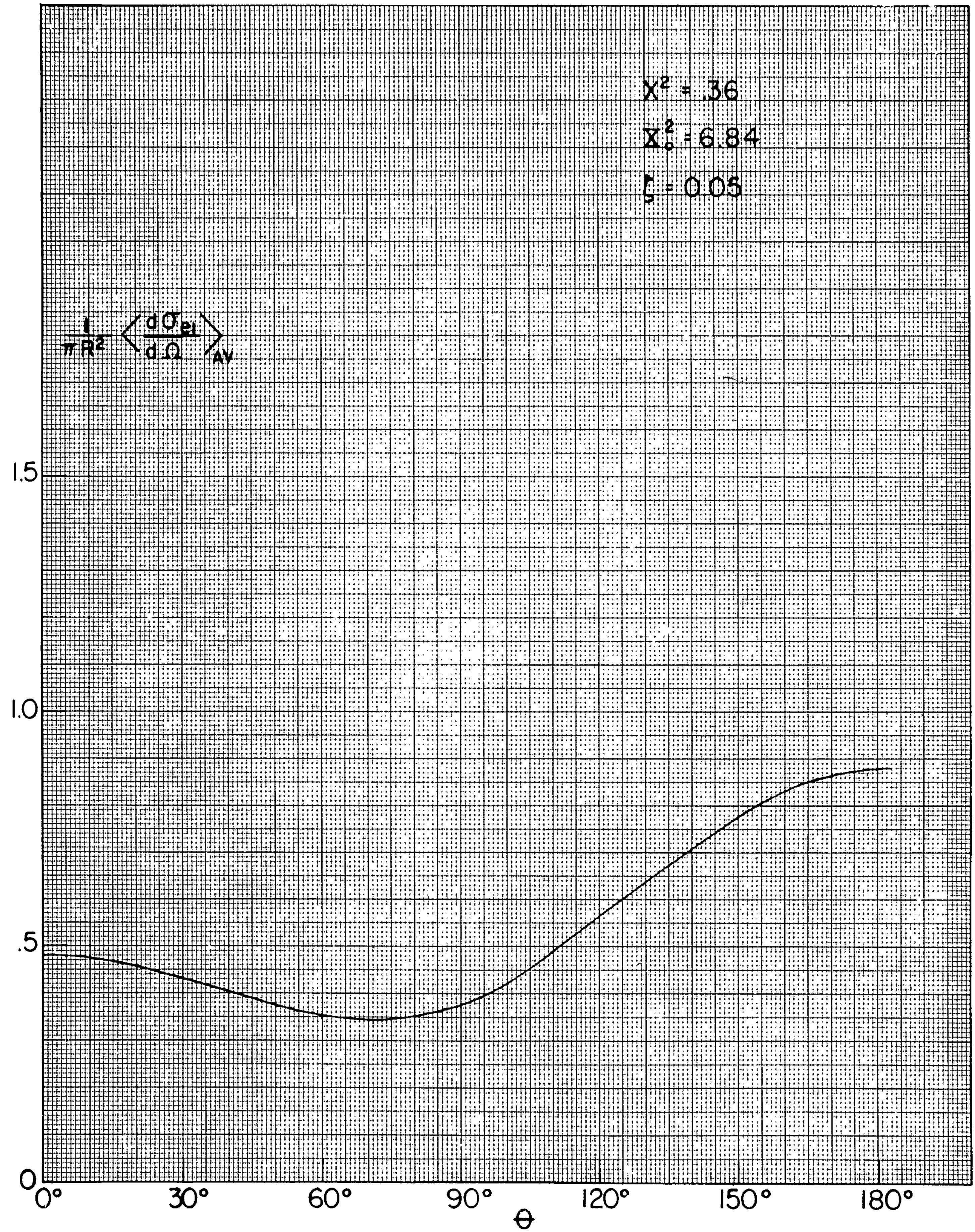


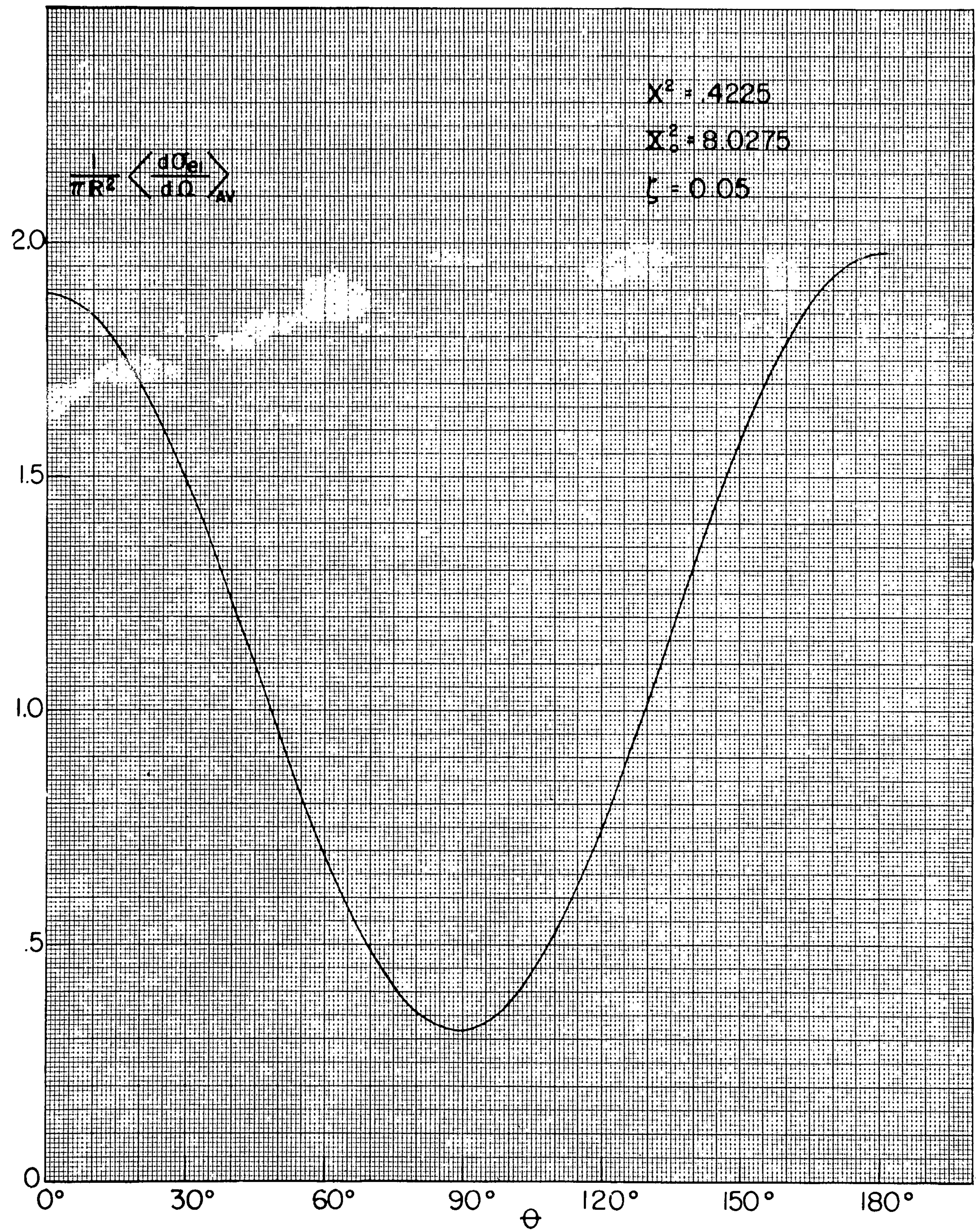
$\chi^2 = 4.41$
$\chi_0^2 = 83.79$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \frac{d\sigma_{el}}{d\Omega}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

V. PLOTS OF DIFFERENTIAL CROSS SECTION FOR ELASTIC SCATTERING (SHAPE-ELASTIC + COMPOUND ELASTIC) FOR $x^2/X_o^2 = 1/19$ WITH $\zeta = 0.05$

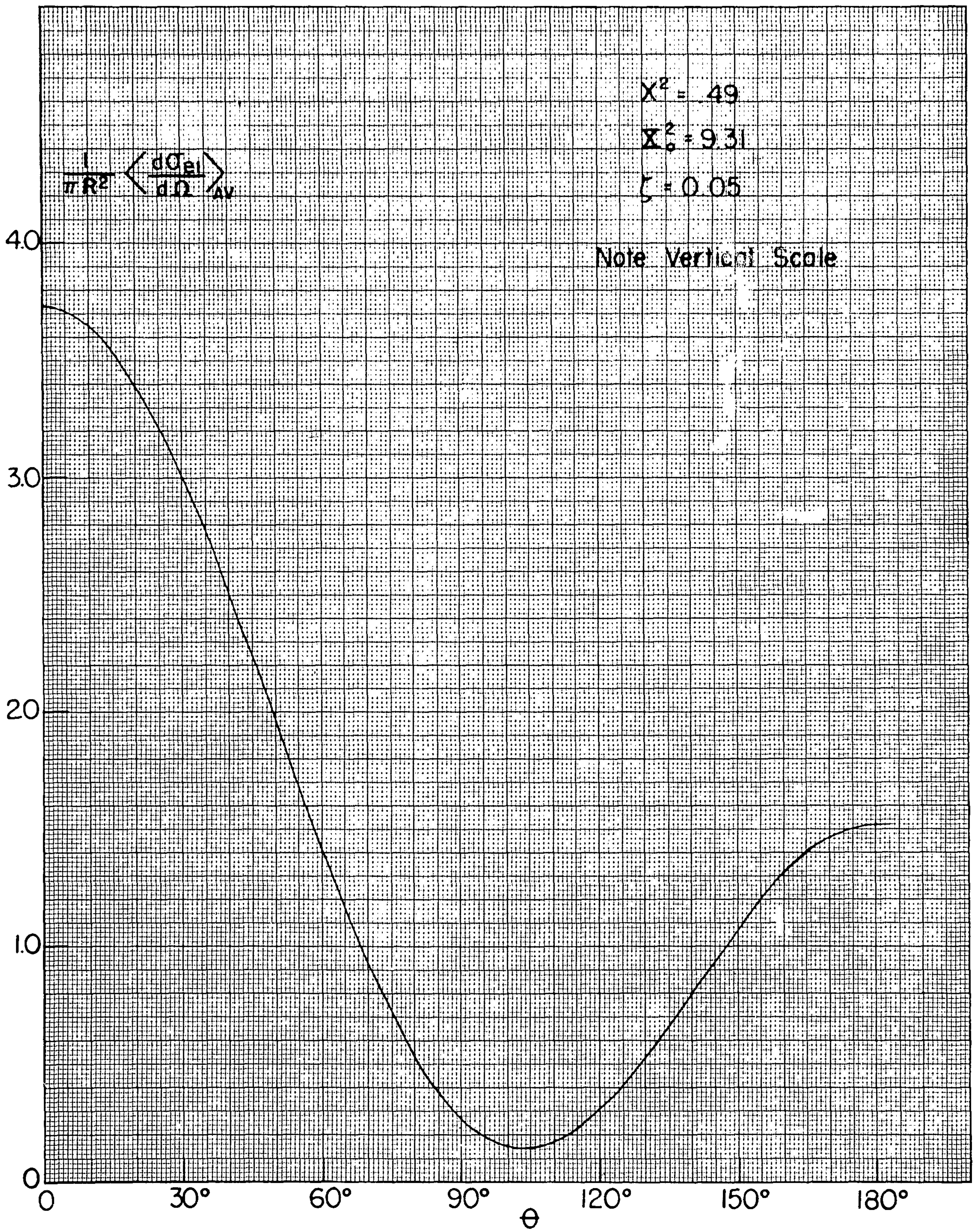


$X^2 = 2.5$
$X_0^2 = 4.75$
$\zeta = 0.05$
$\frac{1}{\pi R^2}\left\langle\frac{d\sigma_{el}}{d\Omega}\right\rangle_{AV}$
1.5
1.0
.5
0
0
30°
60°
90°
120°
150°
180°
θ

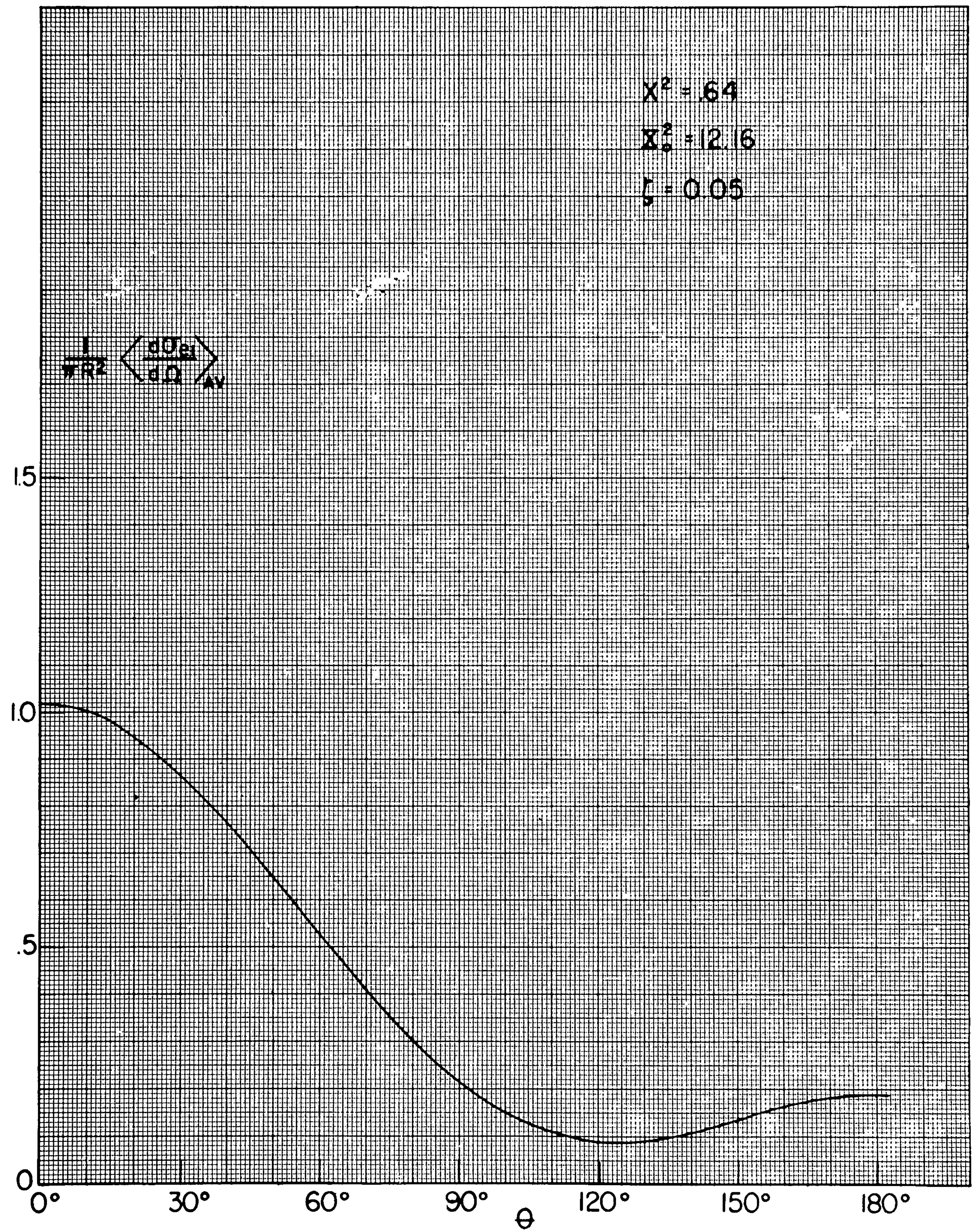

$\chi^2 = .36$
$\chi_0^2 = 6.84$
$\zeta = 0.05$
$\frac{1}{\pi R^2}\left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



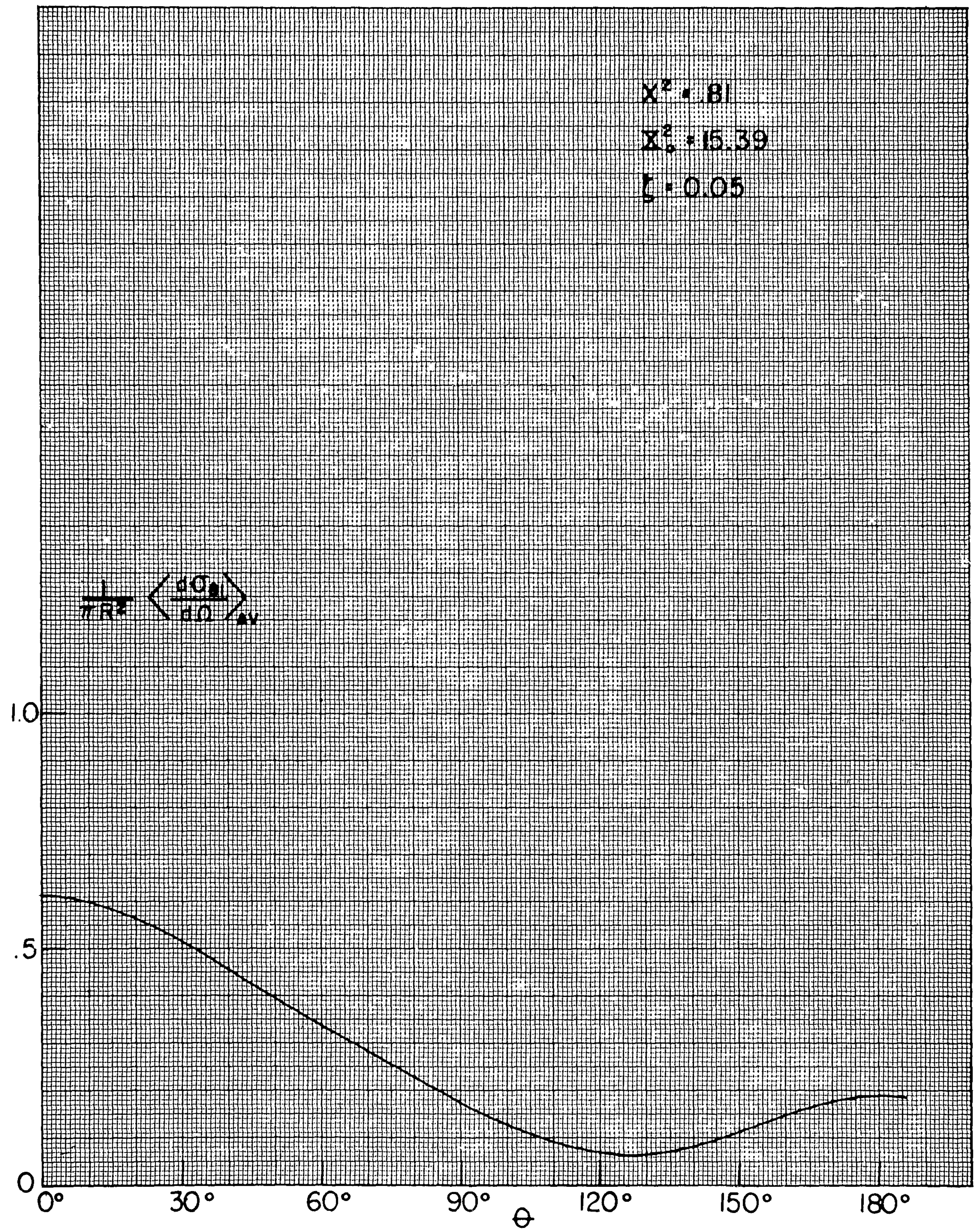
$X^2 = .4225$
$X_0^2 = 8.0275$
$\zeta = 0.05$
$\frac{1}{\pi R^2}\left\langle\frac{d\sigma_{el}}{d\Omega}\right\rangle_{AV}$
2.0
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



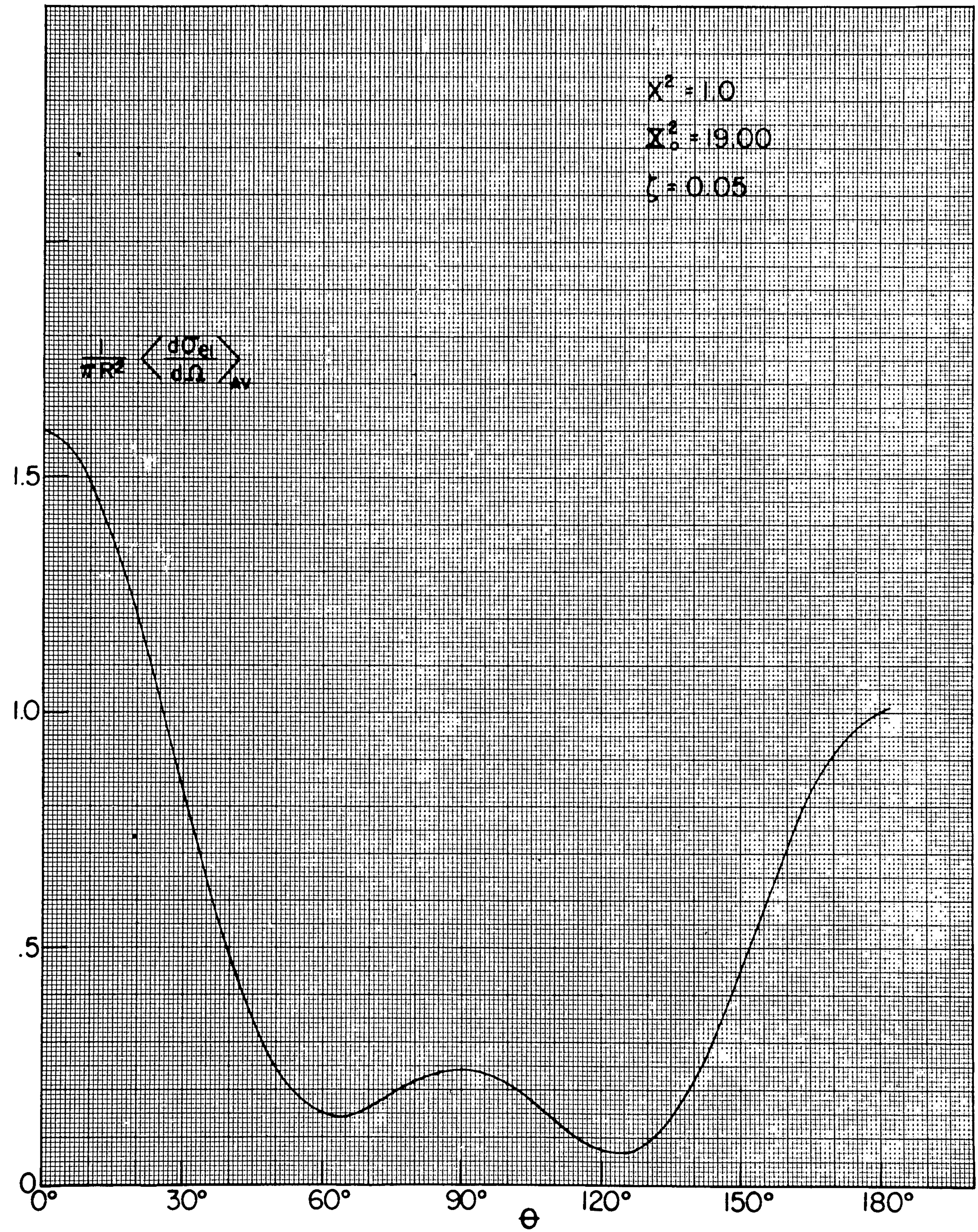
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
$X^2 = .49$
$X_0^2 = 9.31$
$\zeta = 0.05$
Note Vertical Scale
4.0
3.0
2.0
1.0
0
0
30°
60°
90°
120°
150°
180°
θ



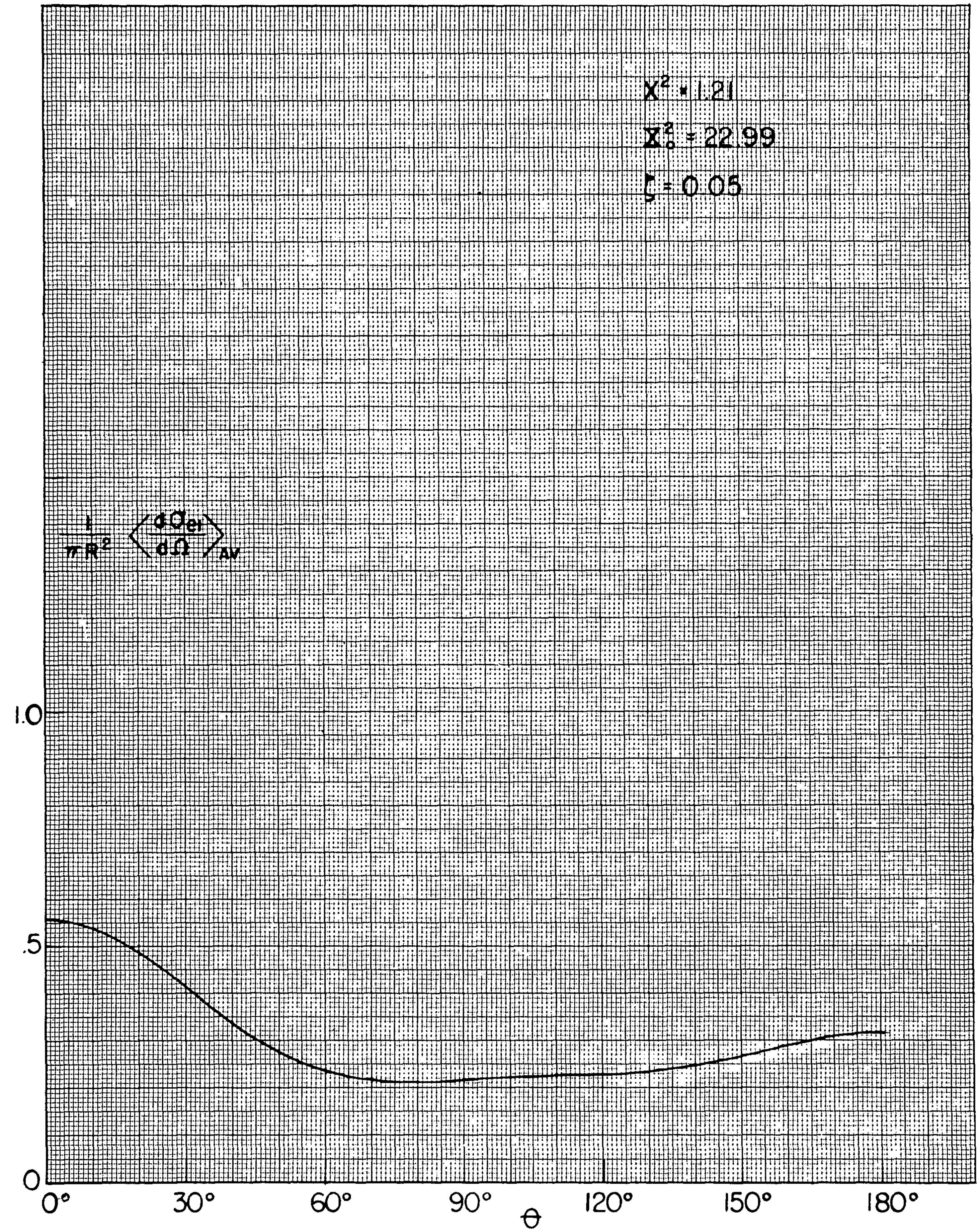
$X^2 = .64$
$X_0^2 = 12.16$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

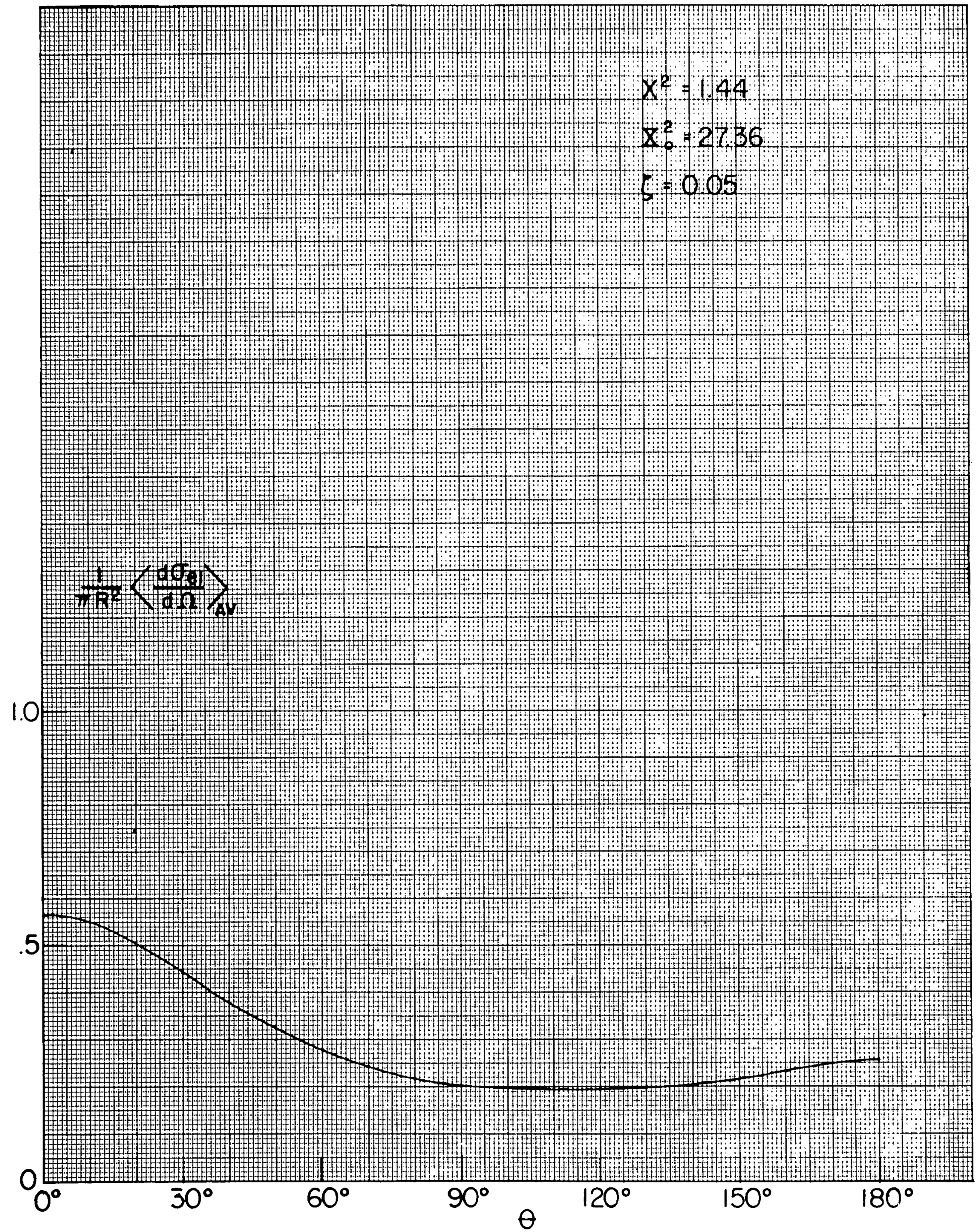


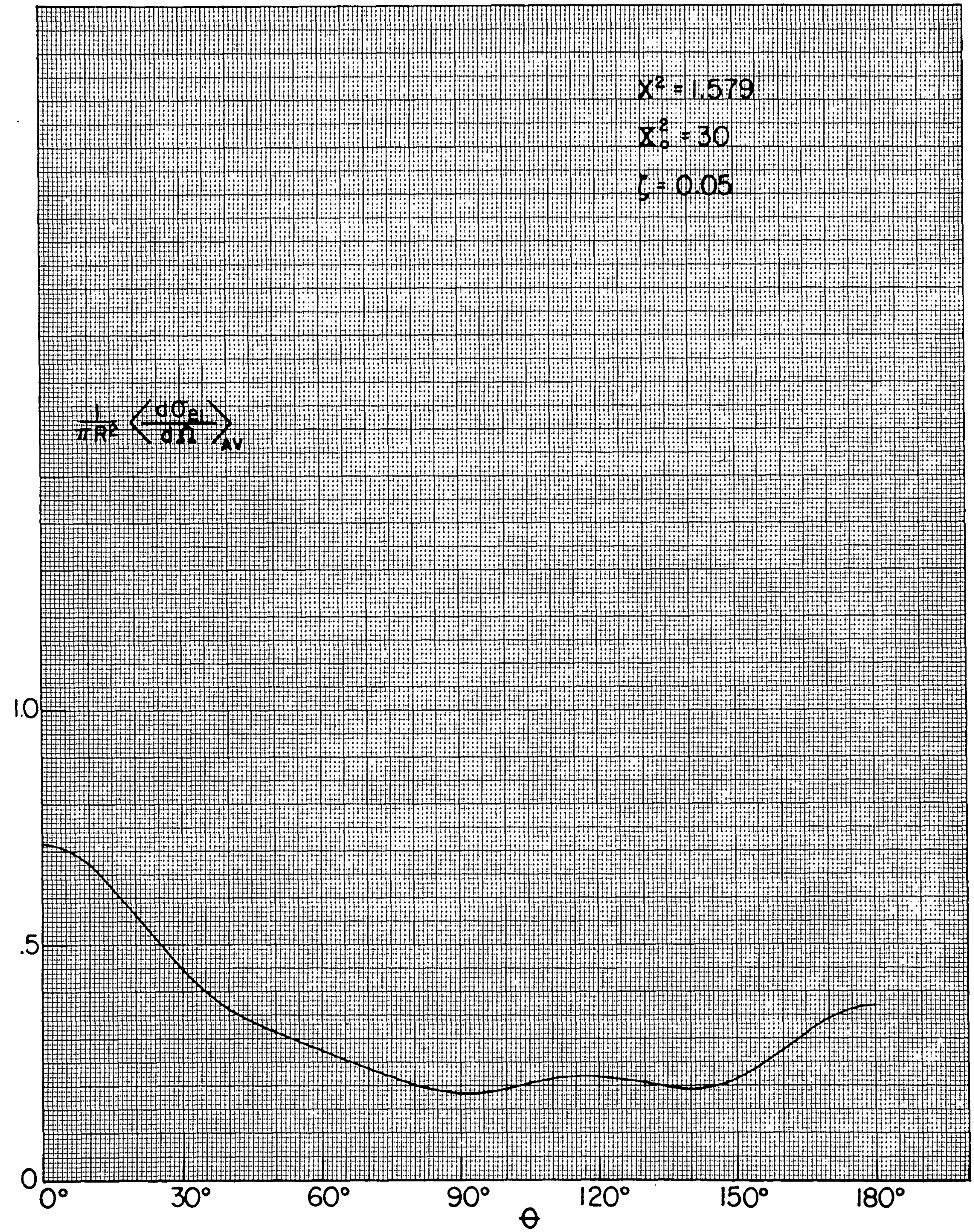
$\chi^2 = .81$
$\chi_0^2 = 15.39$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



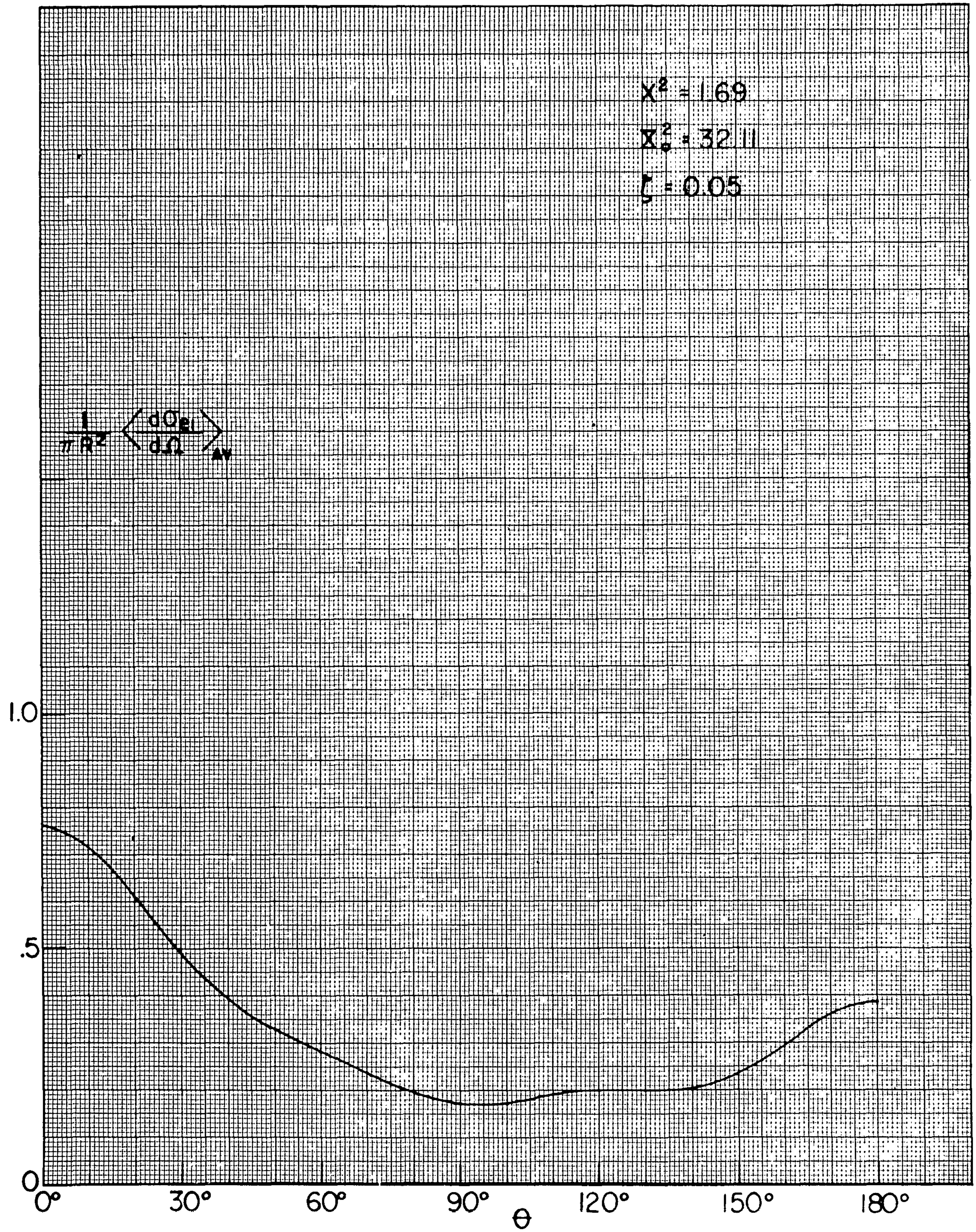
$X^2 = 1.0$
$X_0^2 = 19.00$
$\zeta = 0.05$
$\frac{1}{\pi R^2}\left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

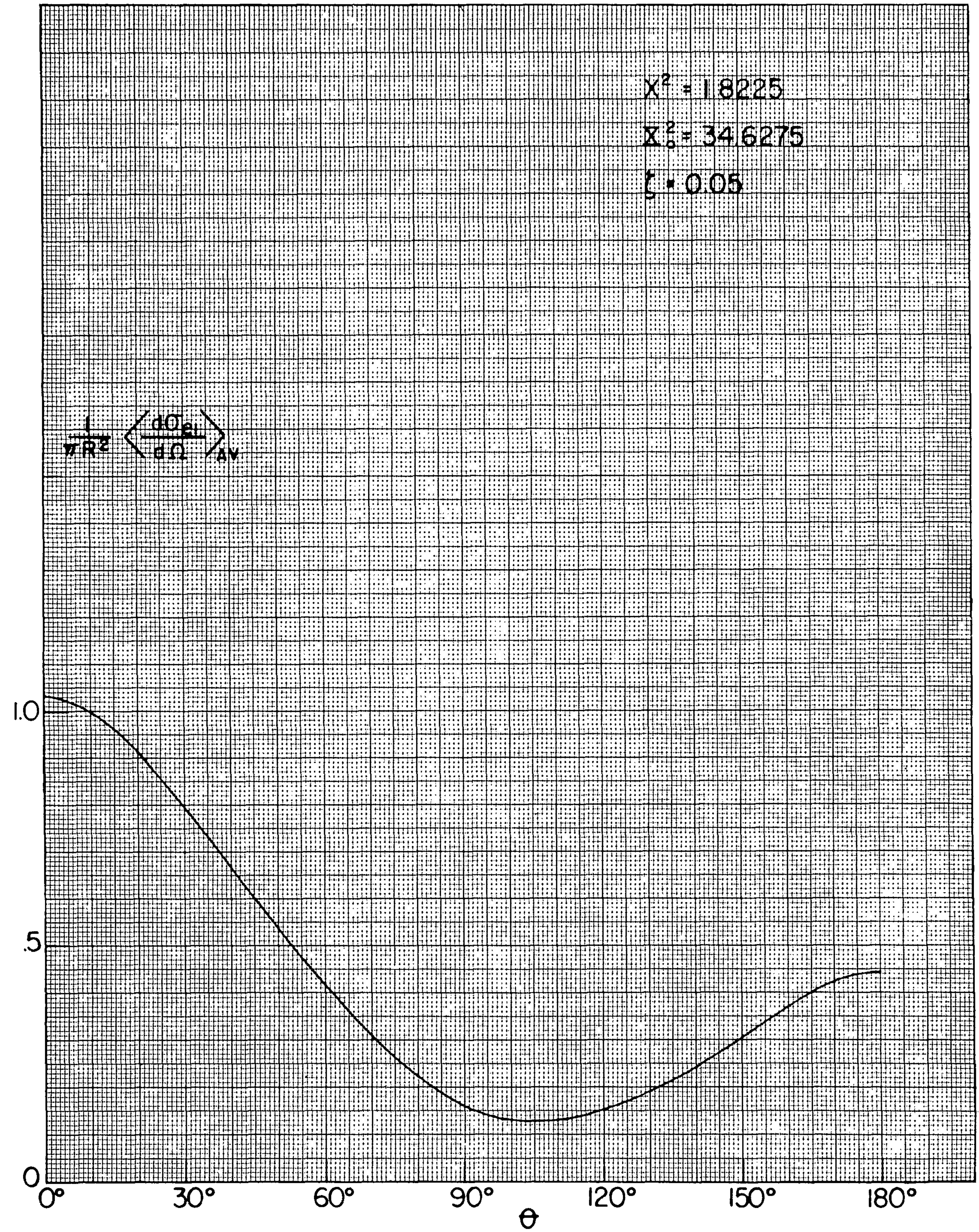

$\chi^2 = 1.21$
$\chi_0^2 = 22.99$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

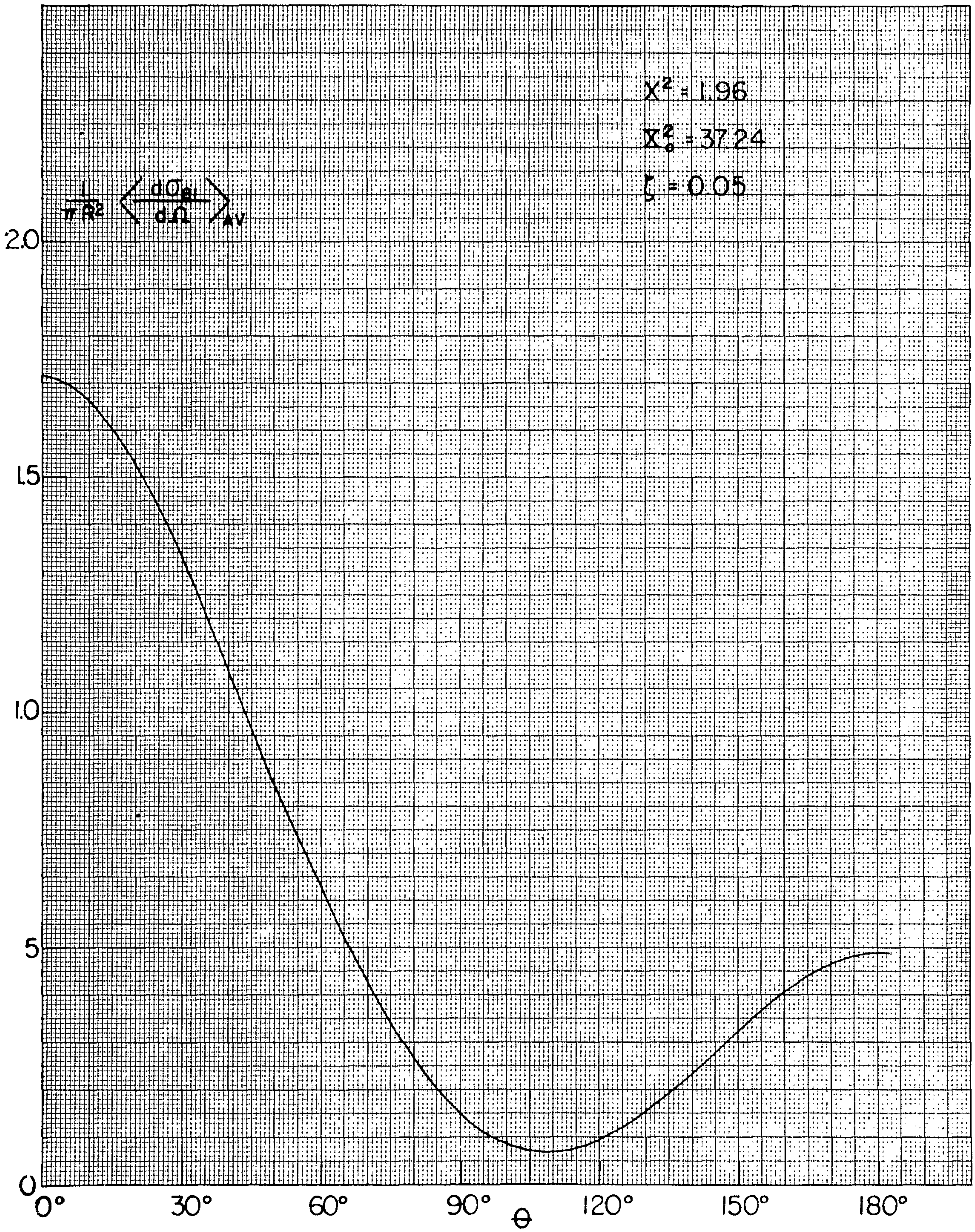

$X^2 = 1.44$
$X_0^2 = 27.36$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



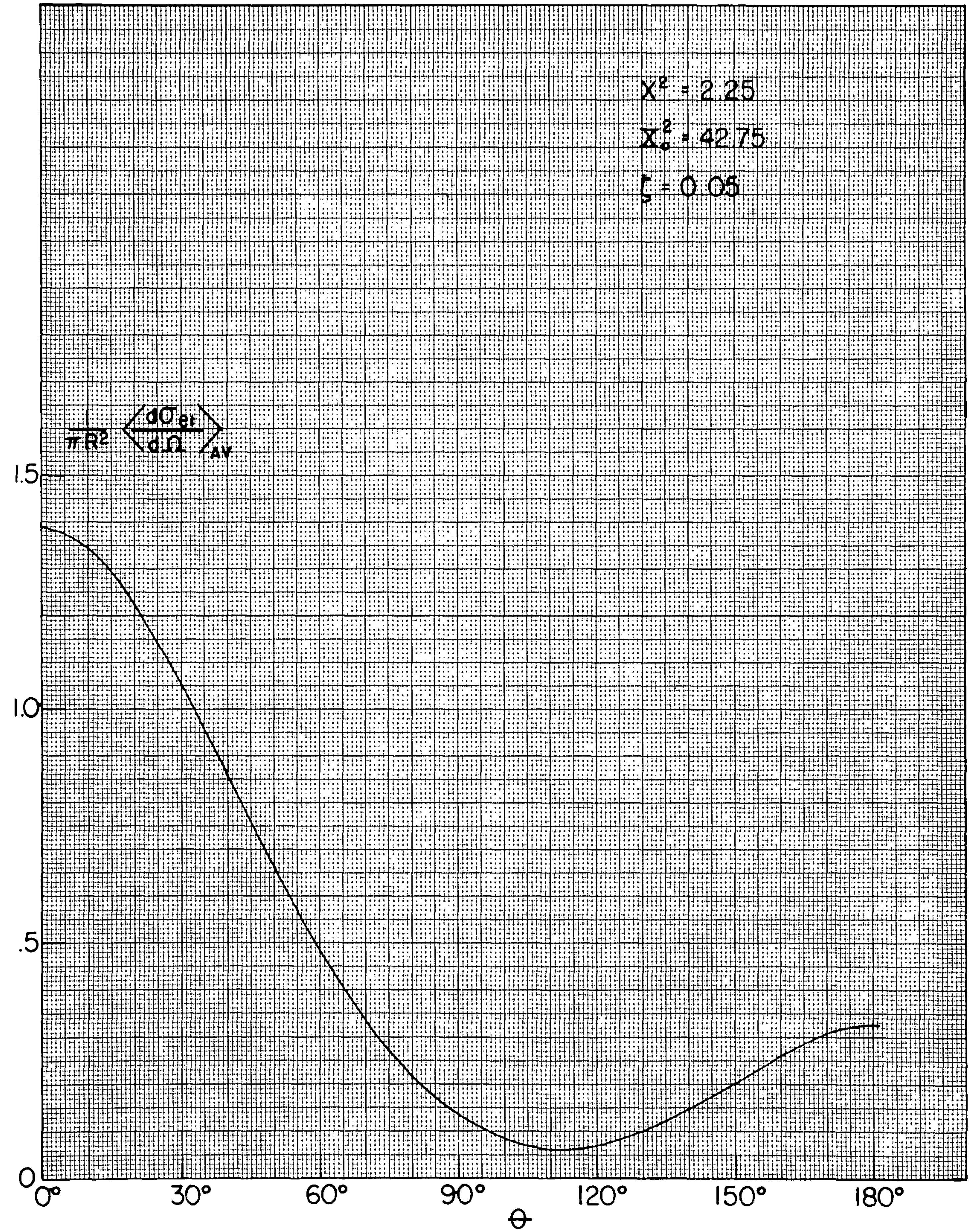
$X^2 = 1.579$
$X_o^2 = 30$
$\zeta = 0.05$
$\frac{1}{\pi R^2}\left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ


$\chi^2 = 1.69$
$\chi_o^2 = 32.11$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

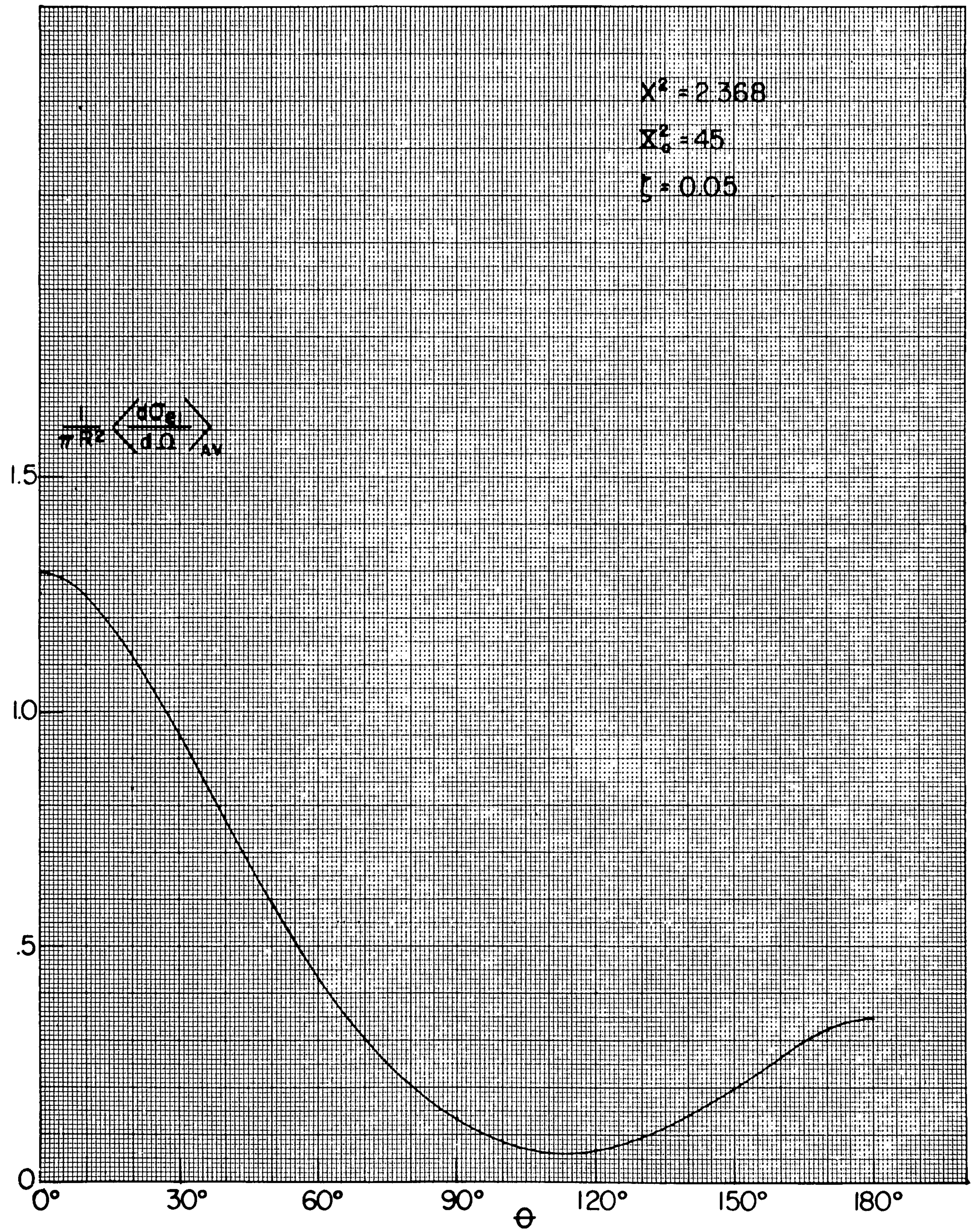

$\chi^2 = 1.8225$
$\bar{\chi}_0^2 = 34.6275$
$\zeta = 0.05$
$\frac{1}{\pi R^2}\left\langle\frac{d\sigma_{el}}{d\Omega}\right\rangle_{AV}$
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



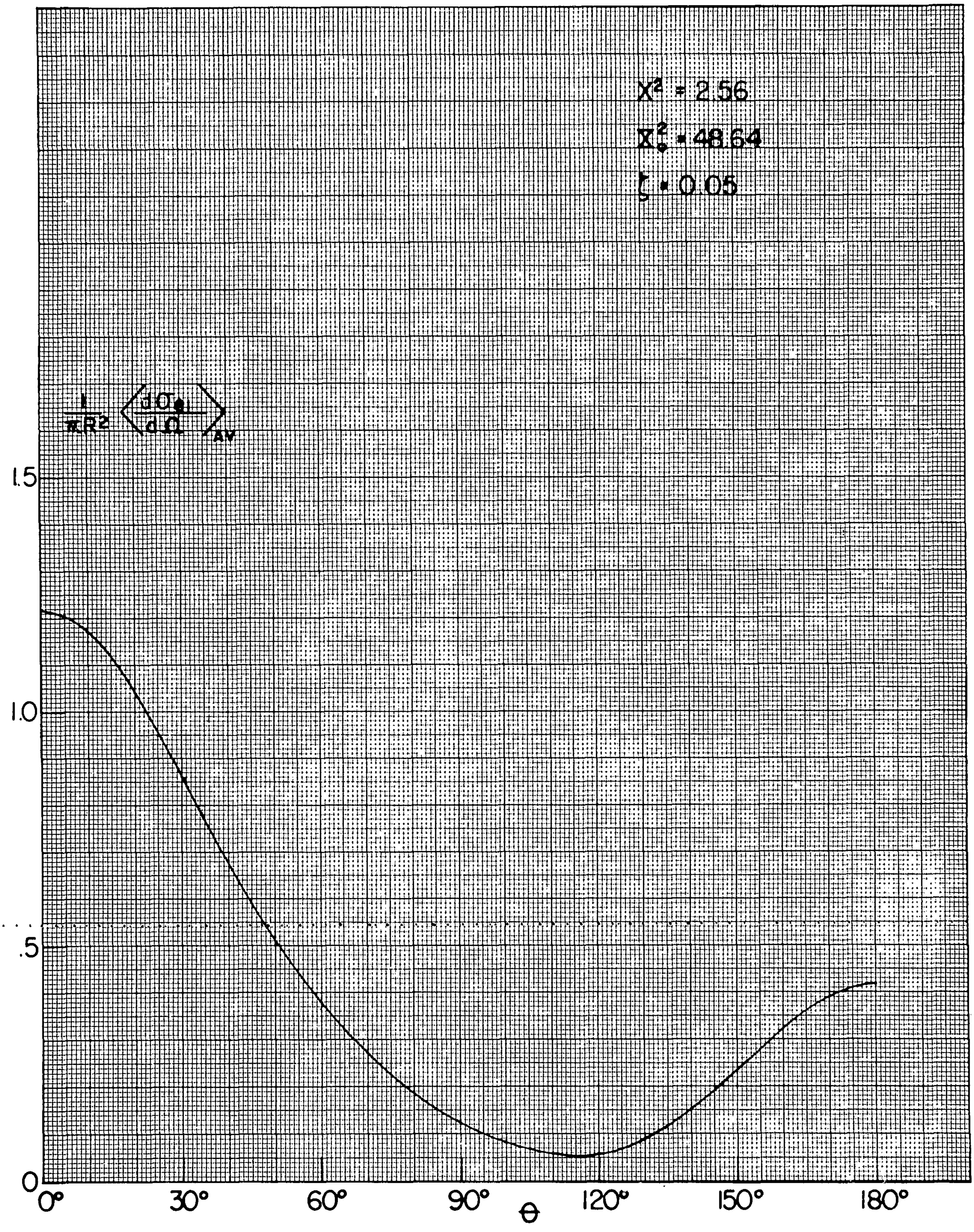
$X^2 = 1.96$
$X_0^2 = 37.24$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
2.0
1.5
1.0
.5
0
0°
30°
60°
90°
θ
120°
150°
180°



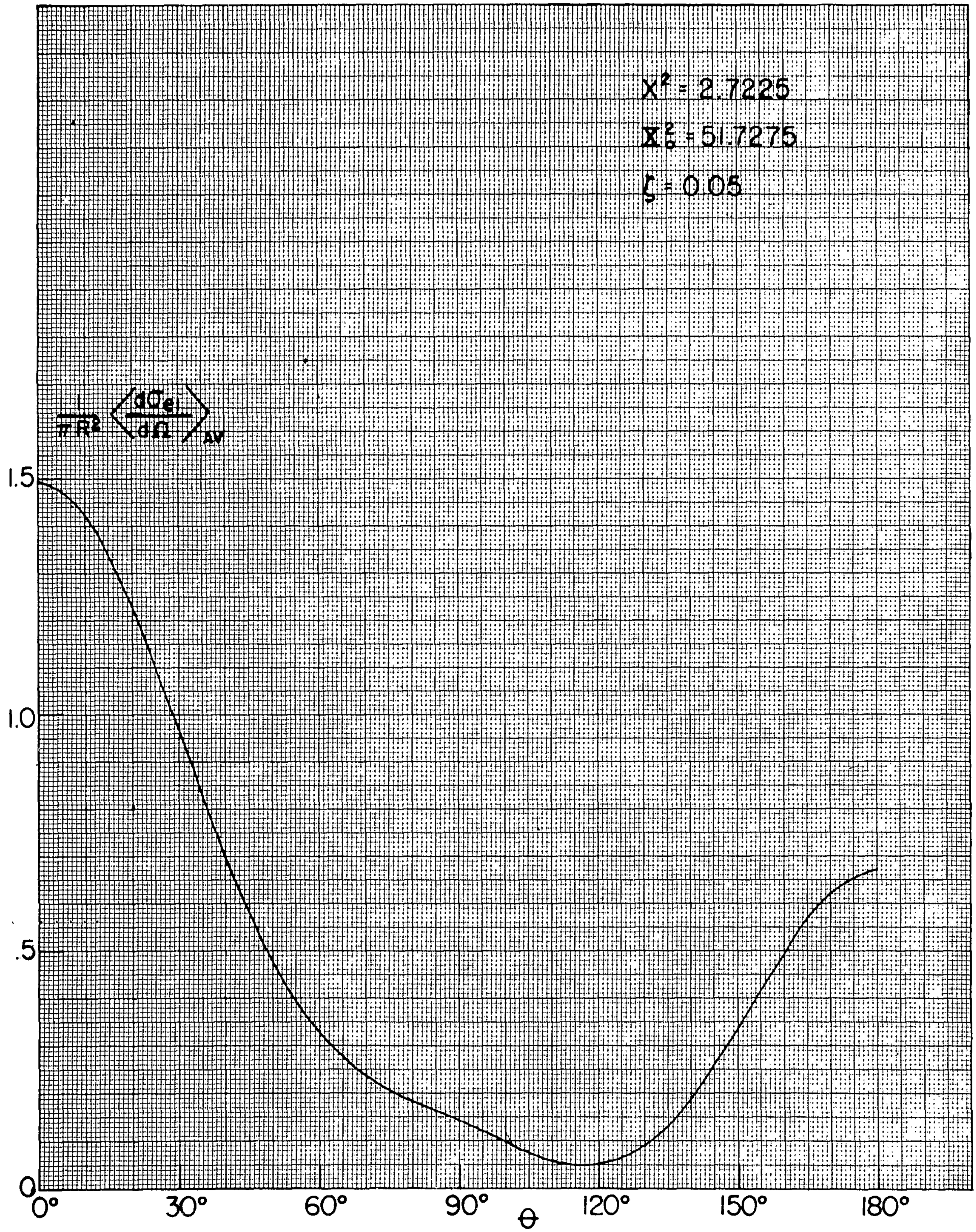
X² = 2.25
X₀² = 42.75
ζ = 0.05
1/πR² ⟨dσ_el/dΩ⟩_AV
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



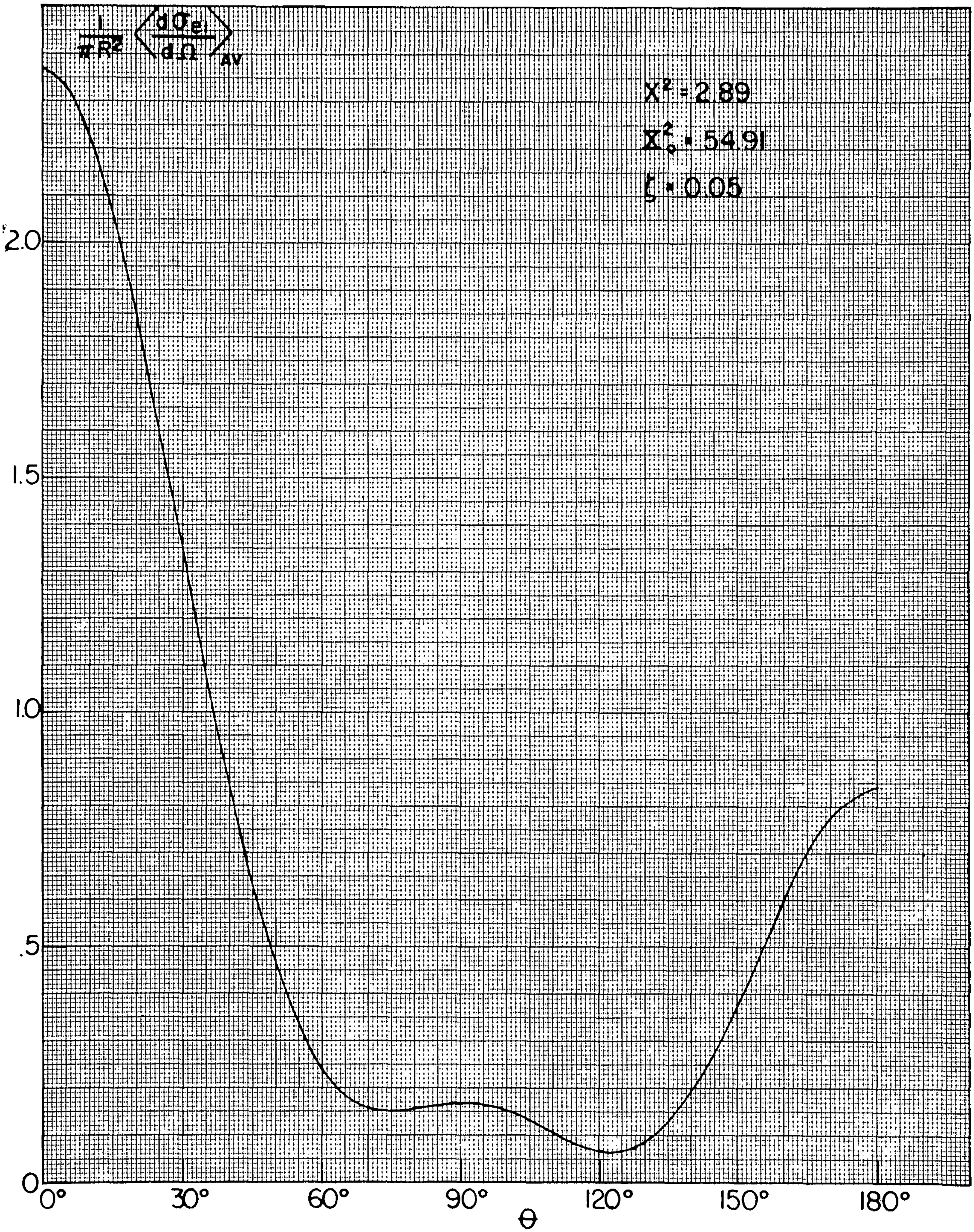
$X^2 = 2.368$
$X_0^2 = 45$
$\zeta = 0.05$
$\frac{1}{\pi R^2}\left\langle\frac{d\sigma_s}{d\Omega}\right\rangle_{AV}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



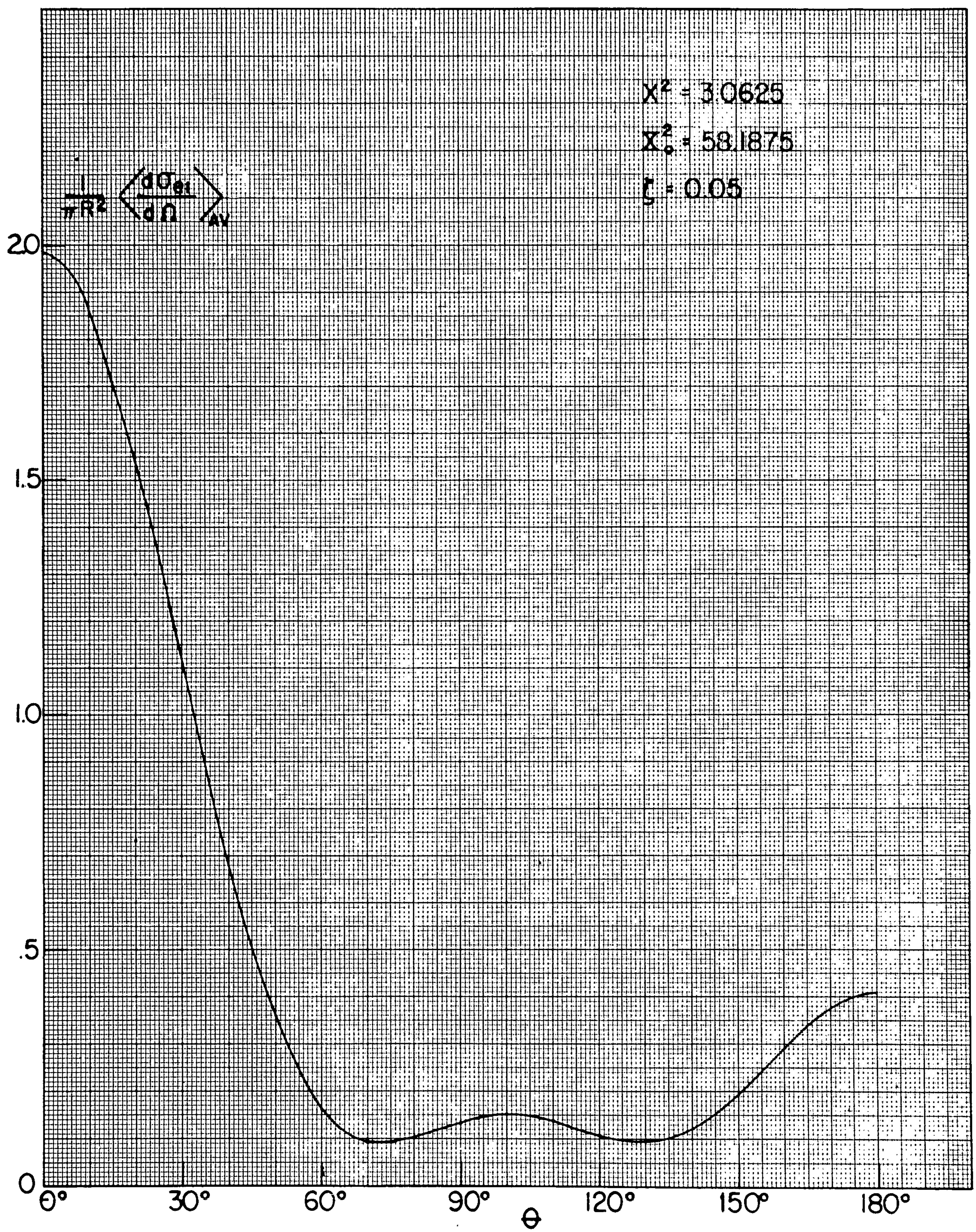
$\chi^2 = 2.56$
$\chi_o^2 = 48.64$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

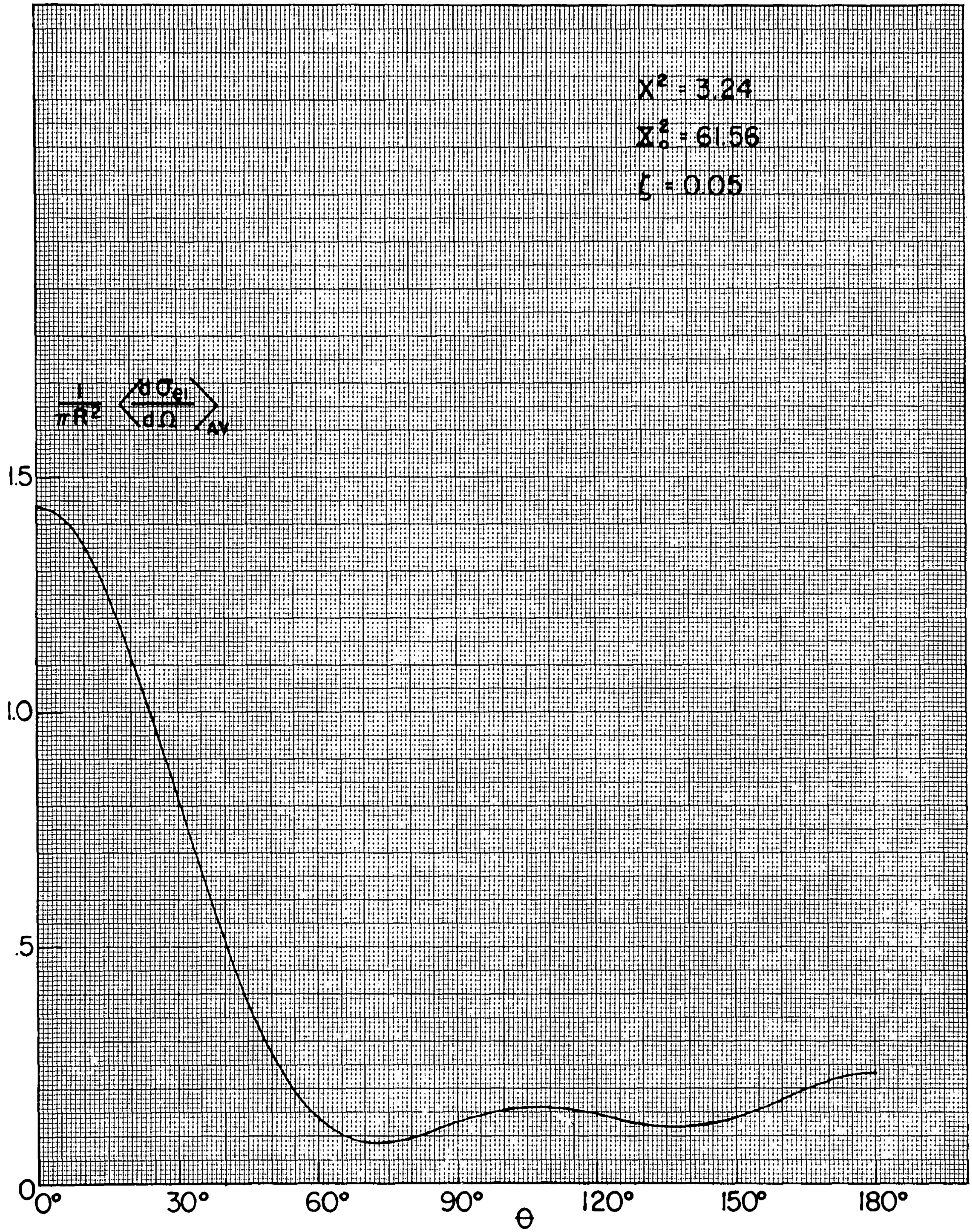


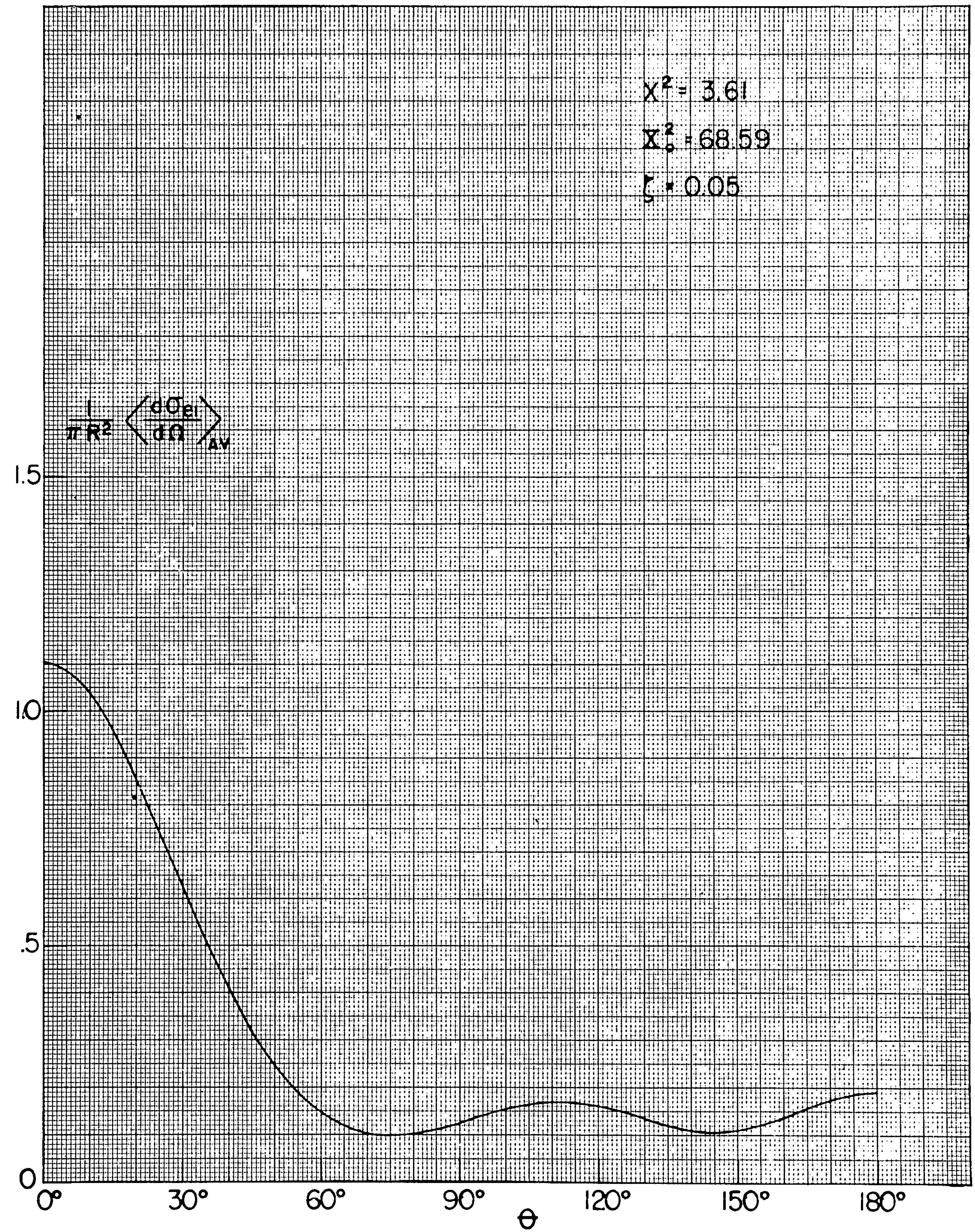
$X^2 = 2.7225$
$X_0^2 = 51.7275$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



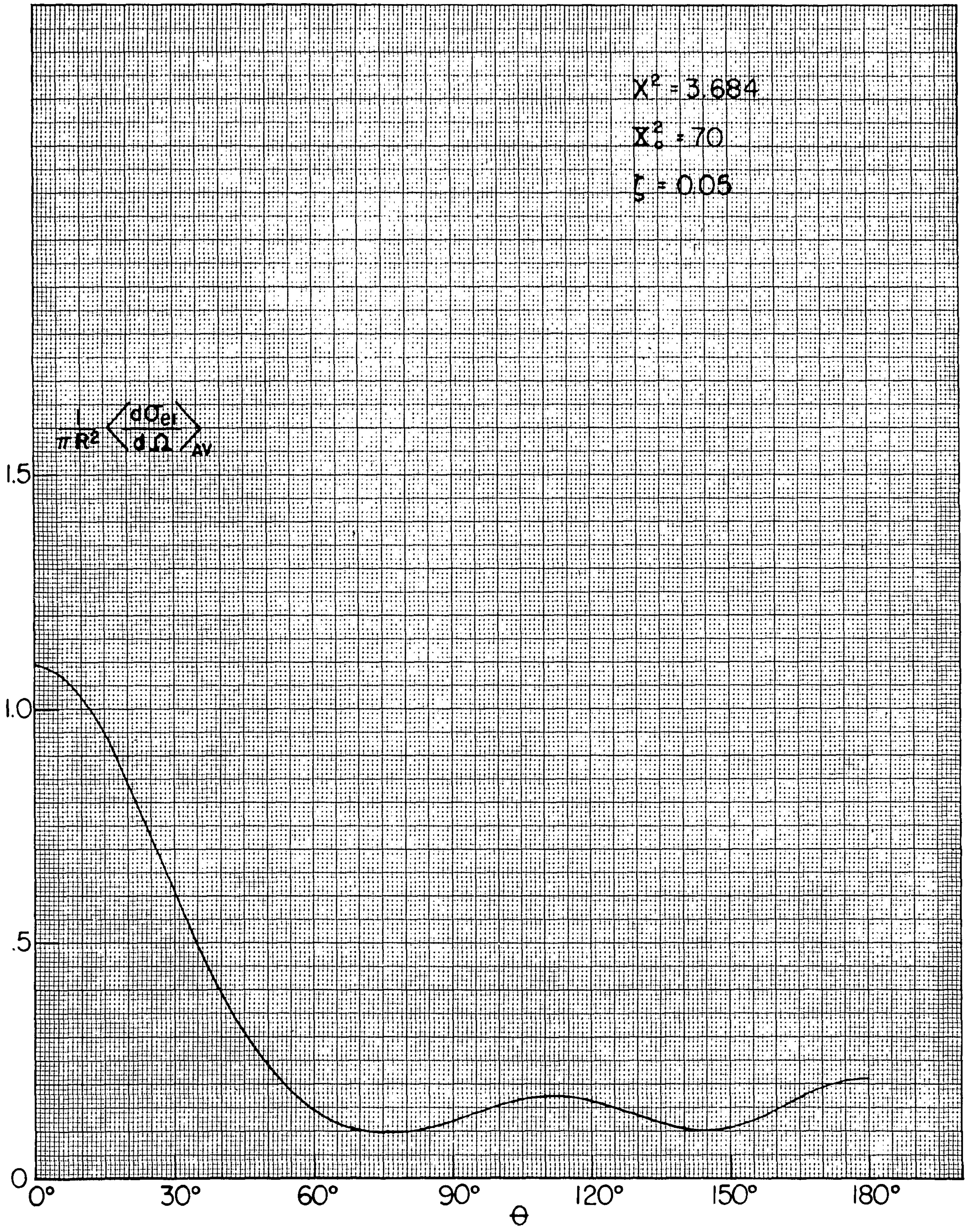
$\frac{1}{\pi R^2}\left\langle\frac{d\sigma_{el}}{d\Omega}\right\rangle_{AV}$
$X^2 = 2.89$
$X_0^2 = 54.91$
$\zeta = 0.05$
2.0
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ


$X^2 = 3.0625$
$X_0^2 = 58.1875$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
2.0
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

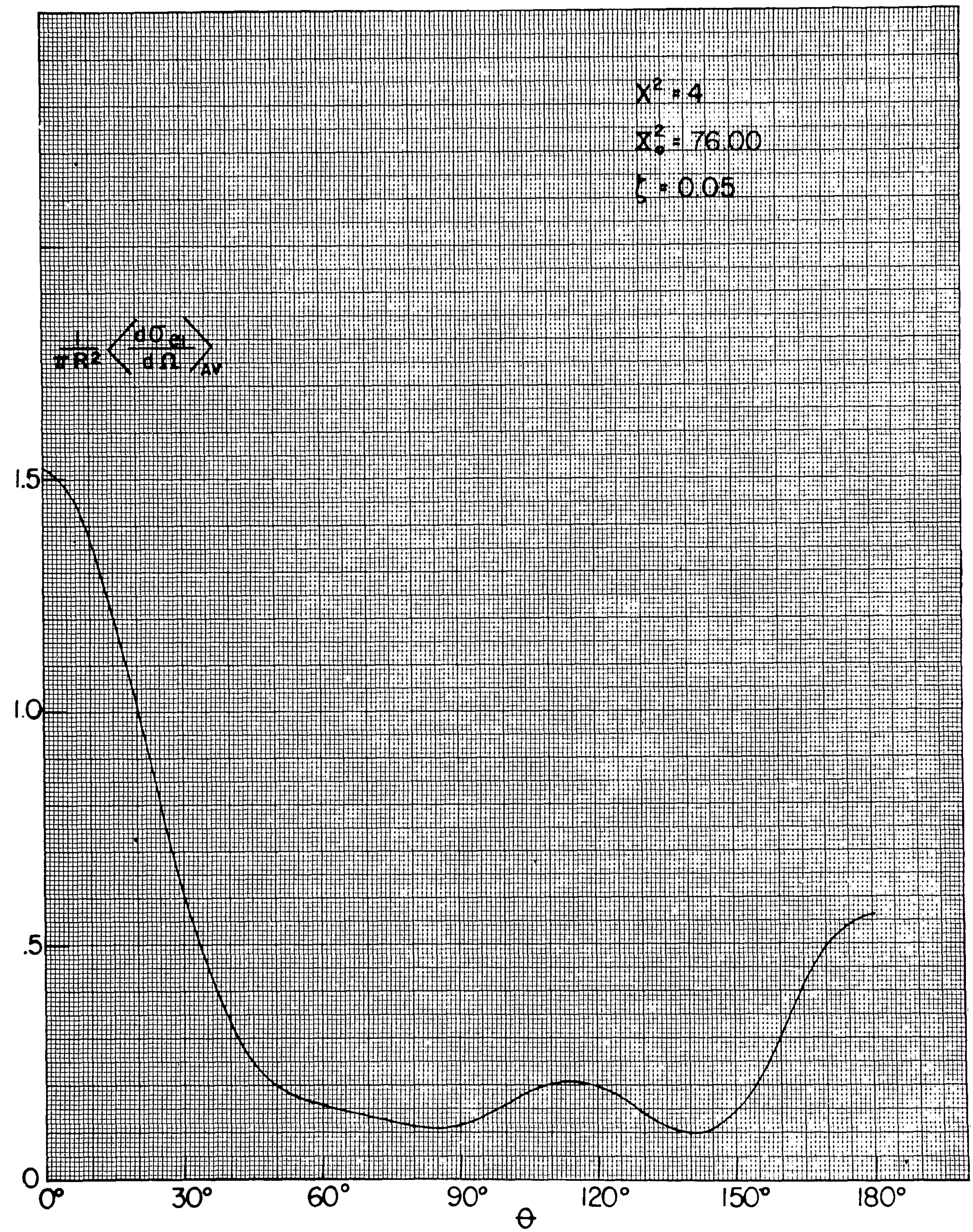

X² = 3.24
X₀² = 61.56
ζ = 0.05
1/πR² ⟨dσ_el/dΩ⟩_AV
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

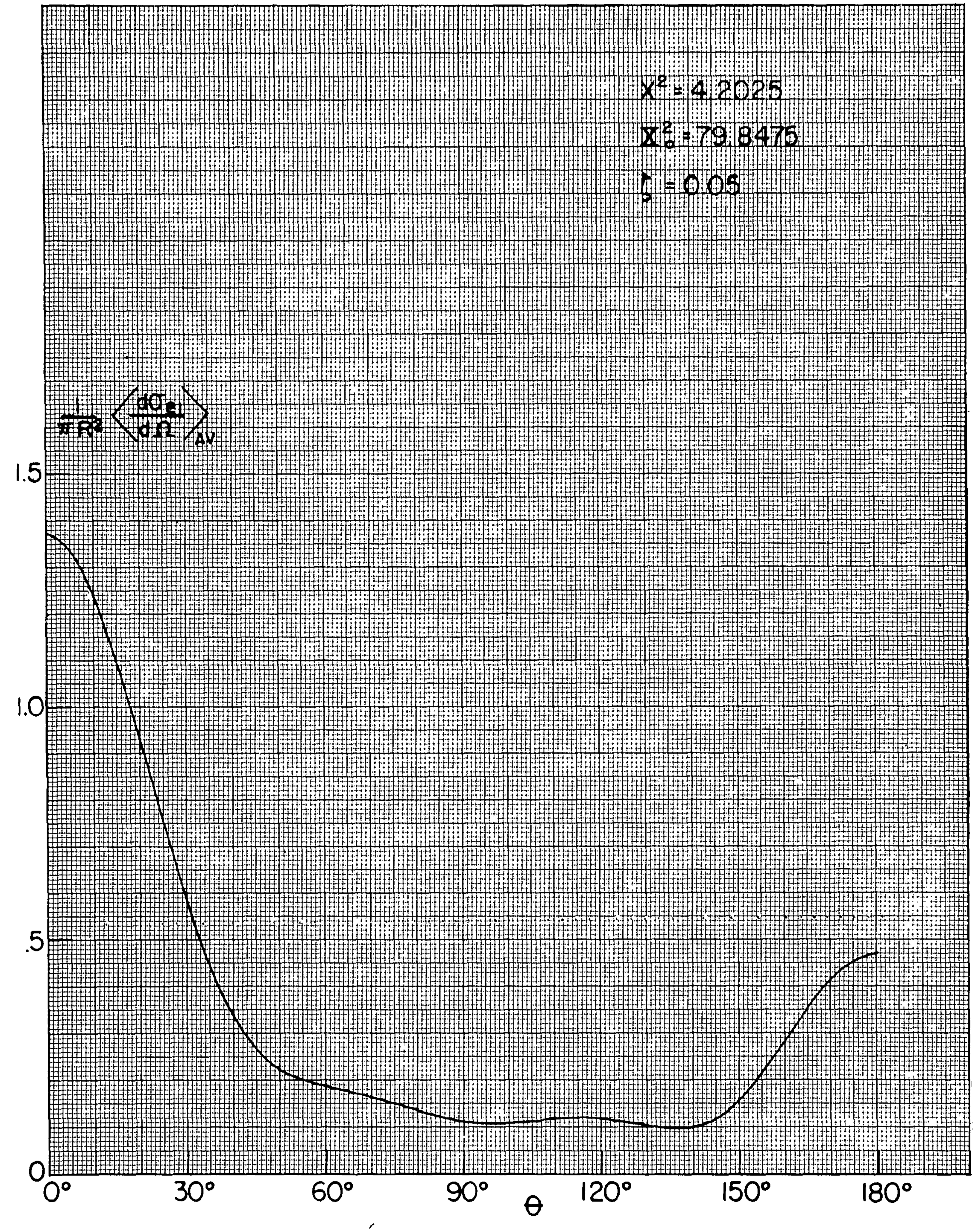


$X^2 = 3.61$
$X_0^2 = 68.59$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

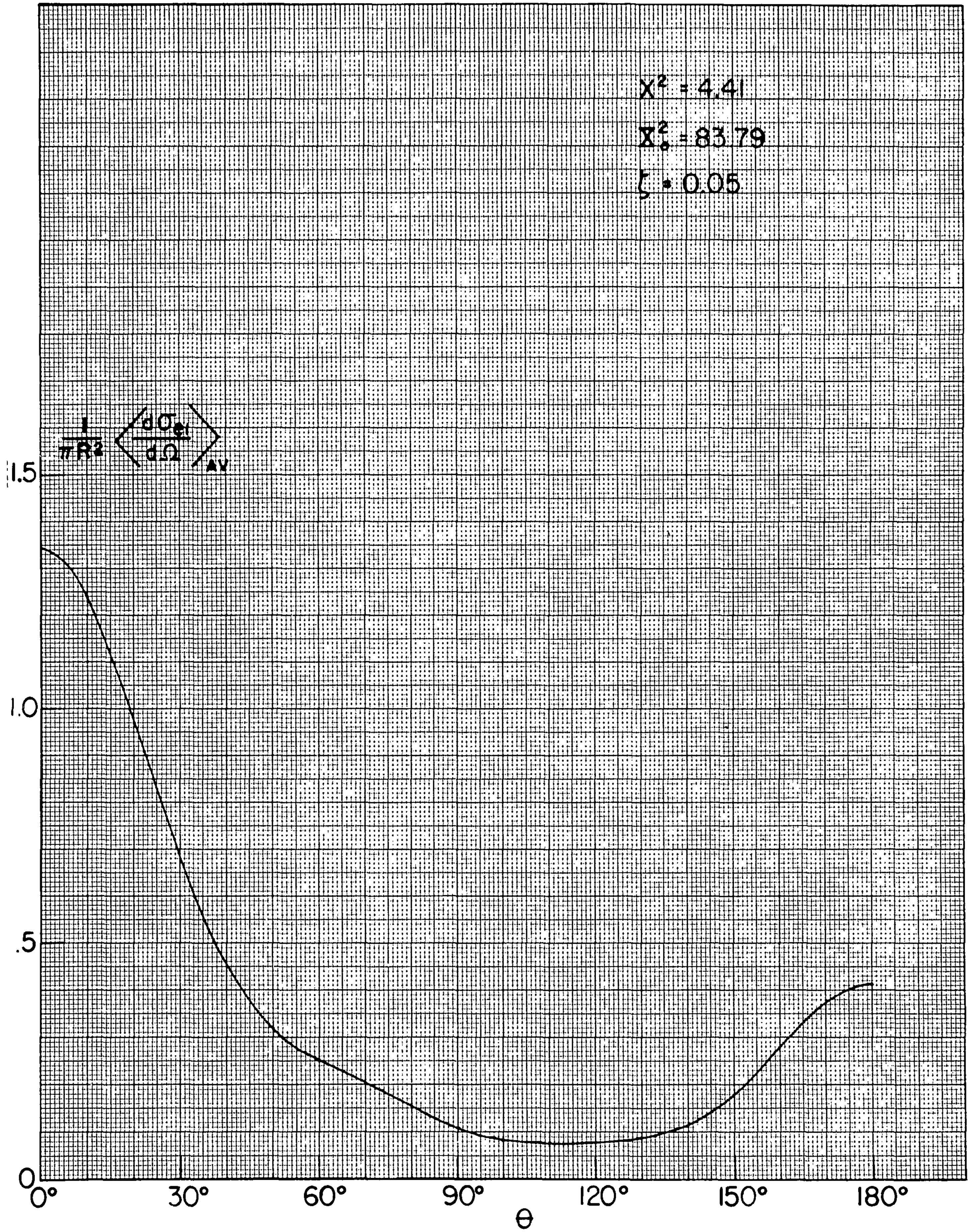


X² = 3.684
X₀² = 70
ζ = 0.05
1/πR² ⟨dσ_el/dΩ⟩_AV
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ


$X^2 = 4$
$X_0^2 = 76.00$
$\zeta = 0.05$
$\frac{1}{\pi R^2}\left\langle\frac{d\sigma_{el}}{d\Omega}\right\rangle_{AV}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ



$\chi^2 = 4.2025$
$\chi_o^2 = 79.8475$
$\zeta = 0.05$
$\frac{1}{\pi R^2}\left\langle\frac{d\sigma_{el}}{d\Omega}\right\rangle_{AV}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ


$X^2 = 4.41$
$X_o^2 = 83.79$
$\zeta = 0.05$
$\frac{1}{\pi R^2} \left\langle \frac{d\sigma_{el}}{d\Omega} \right\rangle_{AV}$
1.5
1.0
.5
0
0°
30°
60°
90°
120°
150°
180°
θ

VI. TABLES OF s_ℓ, Δ_ℓ, $\cos 2\delta_\ell$, $\sin 2\delta_\ell$, AND $\sin^2\delta_\ell$ FOR $x = 0.1(0.1)3.0$ AND $\ell = 0(1)7$

$\ell = 0$

x	s	Δ	cos2δ	sin2δ	sin²δ
.1	= x	= 0	.98007	.19867	.0099667
.2			.92106	.38942	.039470
.3			.82534	.56464	.087332
.4			.69671	.71736	.15165
.5			.54030	.84147	.22984
.6			.36236	.93204	.31882
.7			.16997	.98545	.41502
.8			-.029200	.99957	.51460
.9			-.22720	.97385	.61360
1.0			-.41615	.90930	.70807
1.1			-.58850	.80850	.79425
1.2			-.73739	.67546	.86870
1.3			-.85689	.51550	.92844
1.4			-.94222	.33499	.97111
1.5			-.98999	.141120	.99500
1.6			-.99829	-.058374	.99915
1.7			-.96680	-.25554	.98340
1.8			-.89676	-.44252	.94838
1.9			-.79097	-.61186	.89548
2.0			-.65364	-.75680	.82682
2.1			-.49026	-.87158	.74513
2.2			-.30733	-.95160	.65367
2.3			-.112153	-.99369	.55608
2.4			.087499	-.99616	.45625
2.5			.28366	-.95892	.35817
2.6			.46852	-.88345	.28366
2.7			.63469	-.77276	.18265
2.8			.77557	-.63127	.112217
2.9			.88552	-.46460	.057240
3.0			.96017	-.27942	.019915

$\ell = 1$

x	s	Δ	$\cos 2\delta$	$\sin 2\delta$	$\sin^2\delta$	
.1	.0009901	-.99010	1.000000	.000663	.1098	(-6)
.2	.007692	-.96154	.99999	.005209	6.7831	(-6)
.3	.024771	-.91743	.99985	.017086	.072985	(-3)
.4	.055172	-.86207	.99924	.038977	.37995	(-3)
.5	.100000	-.80000	.99736	.072641	1.32091	(-3)
.6	.15882	-.73529	.99291	.118879	3.5456	(-3)
.7	.23020	-.67114	.98410	.17760	7.9487	(-3)
.8	.31219	-.60976	.96878	.24790	.015608	
.9	.40277	-.55249	.94462	.32817	.027691	
1.0	.50000	-.50000	.90930	.41615	.045351	
1.1	.60226	-.45249	.86076	.50901	.069621	
1.2	.70820	-.40983	.79736	.60350	.101319	
1.3	.81673	-.37175	.71805	.69599	.140975	
1.4	.92702	-.33784	.62247	.78265	.18877	
1.5	1.03846	-.30769	.51100	.85958	.24450	
1.6	1.15056	-.28090	.38498	.92292	.30751	
1.7	1.26298	-.25707	.24638	.96917	.37681	
1.8	1.37547	-.23585	.098034	.99518	.45098	
1.9	1.48785	-.21692	-.056767	.99839	.52838	
2.0	1.6000	-.20000	-.21326	.97700	.60663	
2.1	1.7118	-.18484	-.36762	-.92998	.68381	
2.2	1.8233	-.17123	-.51488	.85726	.75744	
2.3	1.9343	-.15898	-.65021	.75975	.82511	
2.4	2.0450	-.14793	-.76895	.63931	.88447	
2.5	2.1552	-.13793	-.86674	.49876	.93337	
2.6	2.2649	-.12887	-.93977	.34181	.96989	
2.7	2.3743	-.12063	-.98494	.17290	.99247	
2.8	2.4833	-.11312	-.999996	-.002863	.999998	
2.9	2.5918	-.10627	-.98368	-.17995	.99184	
3.0	2.7000	-.10000	-.93579	-.35257	.96789	

$\ell = 2$

x	S	Δ	cos 2δ	sin 2δ	sin ²δ
.1	1.10741 (-6)	-1.9967	1.000000	.000000	.000000
.2	.035082(-3)	-1.9865	1.000000	.000014	.000000
.3	.26191 (-3)	-1.9692	1.000000	.000106	.0028 (-6)
.4	1.07726 (-3)	-1.9441	1.000000	.000438	.0479 (-6)
.5	3.1847 (-3)	-1.9108	.999999	.001307	.4268 (-6)
.6	7.6164 (-3)	-1.8688	.999995	.003162	2.4999 (-6)
.7	.015693	-1.8179	.999978	.006611	.0109266(-3)
.8	.028922	-1.7582	.999923	.012397	.038423 (-3)
.9	.048857	-1.6904	.999772	.021364	.114123 (-3)
1.0	.076923	-1.6154	.999408	.034406	.29603 (-3)
1.1	.114269	-1.5347	.99863	.052400	.68691 (-3)
1.2	.16165	-1.4500	.99710	.076149	1.45178 (-3)
1.3	.21936	-1.3630	.99433	.106319	2.8340 (-3)
1.4	.28727	-1.2756	.98967	.143384	5.1665 (-3)
1.5	.36486	-1.1891	.98225	.18759	8.8760 (-3)
1.6	.45132	-1.1053	.97104	.23890	.0144775
1.7	.54564	-1.0249	.95488	.29699	.022560
1.8	.64672	- .9487	.93248	.36122	.033760
1.9	.75348	- .8773	.90253	.43062	.048734
2.0	.86487	- .8108	.86376	.50390	.068120
2.1	.97991	- .7493	.81501	.57944	.092494
2.2	1.09779	- .6928	.75533	.65534	.122335
2.3	1.21776	- .6408	.68404	.71665	.15798
2.4	1.33921	- .5933	.60080	.79940	.19960
2.5	1.46165	- .5392	.50571	.86270	.24715
2.6	1.5847	- .5106	.39931	.91682	.30035
2.7	1.7079	- .4746	.28262	.95923	.35869
2.8	1.8312	- .4416	.15719	.98757	.42140
2.9	1.9542	- .4119	.025032	.99969	.48748
3.0	2.0769	- .3846	-.111374	.99378	.55569

$\ell = 3$

x	s	Δ	cos 2δ	sin 2δ	sin $^2\delta$
.1	.44356 (-9)	-2.9980	1.000000	.000000	.000000
.2	.056435 (-6)	-2.9920	1.000000	.000000	.000000
.3	.95460 (-6)	-2.9819	1.000000	.000000	000000
.4	7.0512 (-6)	-2.9676	1.000000	.000002	000000
.5	.033014 (-3)	-2.9491	1.000000	.000010	000000
.6	.115664 (-3)	-2.9260	1.000000	.000034	.000000
.7	.33126 (-3)	-2.8983	1.000000	.000097	.0023 (-6)
.8	.81754 (-3)	-2.8655	1.000000	.000241	.0145 (-6)
.9	1.7987 (-3)	-2.8273	1.000000	.000534	.0713 (-6)
1.0	3.6101 (-3)	-2.7834	.999999	.001082	.2928 (-6)
1.1	6.7196 (-3)	-2.7333	.999998	.002037	1.0374 (-6)
1.2	.0117394	-2.6768	.999994	.003605	3.2489 (-6)
1.3	.019426	-2.6136	.999982	.006049	9.1484 (-6)
1.4	.030663	-2.5437	.999953	.009701	.023529(-3)
1.5	.046427	-2.4669	.999888	.014949	.055872(-3)
1.6	.067735	-2.3838	.999753	.022236	.123623(-3)
1.7	.095583	-2.2949	.999486	.032045	.25679 (-3)
1.8	.13087	-2.2009	.99899	.044886	.50394 (-3)
1.9	.17435	-2.1028	.99812	.061267	.93930 (-3)
2.0	.22655	-2.0018	.99666	.081679	1.6706 (-3)
2.1	.28776	-1.8990	.99431	.106562	2.8470 (-3)
2.2	.35801	-1.7957	.99067	.136286	4.6652 (-3)
2.3	.43708	-1.6932	.98525	.17112	7.3747 (-3)
2.4	.52455	-1.5925	.97744	.21120	.0112790
2.5	.62305	-1.4914	.96653	.25654	.016734
2.6	.72209	-1.4004	.95172	.30697	.024140
2.7	.83063	-1.3102	.93213	.36212	.033934
2.8	.94456	-1.2248	.90685	.42145	.046575
2.9	1.06307	-1.1439	.87496	.48420	.062521
3.0	1.18537	-1.0683	.83557	.54939	.082216

$\ell = 4$

x	s		Δ	cos2δ	sin2δ	$\sin^2\delta$	
.1	.090573	(-12)	-3.9986	1.000000	.000000	.000000	
.2	.046175	(-9)	-3.9943	1.000000	.000000	.000000	
.3	1.7625	(-9)	-3.9871	1.000000	.000000	.000000	
.4	.023239	(-6)	-3.9770	1.000000	.000000	.000000	
.5	.17092	(-6)	-3.9640	1.000000	.000000	.000000	
.6	.86802	(-6)	-3.9480	1.000000	.000000	.000000	
.7	3.4110	(-6)	-3.9290	1.000000	.000000	.000000	
.8	.0111006	(-3)	-3.9068	1.000000	.000000	.000000	
.9	.031256	(-3)	-3.8813	1.000000	.000002	.000000	
1.0	.078456	(-3)	-3.8526	1.000000	.000007	.000000	
1.1	.17934	(-3)	-3.8203	1.000000	.000018	.000000	
1.2	.37920	(-3)	-3.7843	1.000000	.000041	.000000	
1.3	.75056	(-3)	-3.7445	1.000000	.000088	.0019	(-6)
1.4	1.40354	(-3)	-3.7005	1.000000	.000175	.0076	(-6)
1.5	2.4977	(-3)	-3.6521	1.000000	.000329	.0271	(-6)
1.6	4.2544	(-3)	-3.5990	1.000000	.000592	.0875	(-6)
1.7	6.9694	(-3)	-3.5410	1.000000	.001018	.2589	(-6)
1.8	.0110227		-3.4777	.999999	.001685	.7097	(-6)
1.9	.016885		-3.4090	.999996	.002696	1.8175	(-6)
2.0	.025121		-3.3345	.999991	.004184	4.3757	(-6)
2.1	.036381		-3.2542	.999980	.006312	9.9596	(-6)
2.2	.051389		-3.1681	.999957	.009280	.021532	(-3)
2.3	.070917		-3.0763	.999911	.013326	.044397	(-3)
2.4	.095760		-2.9790	.999825	.018719	.087608	(-3)
2.5	.127492		-2.8763	.999668	.025763	.16596	(-3)
2.6	.16444		-2.7702	.999395	.034787	.30263	(-3)
2.7	.20961		-2.6600	.99893	.046140	.53251	(-3)
2.8	.26270		-2.5469	.99820	.060008	.90105	(-3)
2.9	.32405		-2.4320	.99702	.077253	1.49424	(-3)
3.0	.39377		-2.3163	.99522	.097702	2.3921	(-3)
				.99255	.121829	3.7244	(-3)

$\ell = 5$

x	S		Δ	cos2δ	sin2δ	sin²δ	
.1	.0111855	(-15)	-4.9989	1.000000	.000000	.000000	
.2	.022832	(-12)	-4.9955	1.000000	.000000	.000000	
.3	1.9639	(-12)	-4.9900	1.000000	.000000	.000000	
.4	.046139	(-9)	-4.9822	1.000000	.000000	.000000	
.5	.53176	(-9)	-4.9721	1.000000	.000000	.000000	
.6	3.9028	(-9)	-4.9597	1.000000	.000000	.000000	
.7	.020964	(-6)	-4.9451	1.000000	.000000	.000000	
.8	.089554	(-6)	-4.9282	1.000000	.000000	.000000	
.9	.32097	(-6)	-4.9088	1.000000	.000000	.000000	
1.0	1.00112	(-6)	-4.8870	1.000000	.000000	.000000	
1.1	2.1892	(-6)	-4.8628	1.000000	.000000	.000000	
1.2	7.0764	(-6)	-4.8361	1.000000	.000000	.000000	
1.3	.016588	(-3)	-4.8067	1.000000	.000003	.000000	
1.4	.036341	(-3)	-4.7747	1.000000	.000007	.000000	
1.5	.075071	(-3)	-4.7399	1.000000	.000014	.000000	
1.6	.147293	(-3)	-4.7022	1.000000	.000028	.000000	
1.7	.27611	(-3)	-4.6617	1.000000	.000053	.000000	
1.8	.49691	(-3)	-4.6178	1.000000	.000096	.0023	(-6)
1.9	.86205	(-3)	-4.5707	1.000000	.000168	.0070	(-6)
2.0	1.44658	(-3)	-4.5201	1.000000	.000283	.0201	(-6)
2.1	2.3548	(-3)	-4.4657	1.000000	.000465	.0542	(-6)
2.2	3.7278	(-3)	-4.4075	1.000000	.000744	.1383	(-6)
2.3	5.7511	(-3)	-4.3450	.999999	.001159	.3357	(-6)
2.4	8.6625	(-3)	-4.2782	.999998	.001764	.7779	(-6)
2.5	.0128411		-4.2067	.999997	.002628	1.7268	(-6)
2.6	.018403		-4.1304	.999993	.003837	3.6816	(-6)
2.7	.026023		-4.0490	.999985	.005498	7.5581	(-6)
2.8	.036117		-3.9624	.999970	.007739	.0149748	(-3)
2.9	.049246		-3.8706	.999943	.010713	.028690	(-3)
3.0	.066015		-3.7734	.999894	.014594	.053250	(-3)

$\ell = 6$

x	s		Δ	cos 2δ	sin 2δ	sin $^2\delta$	
.1	.92461	(-21)	-5.9991	1.000000	.000000	.000000	
.2	7.3902	(-18)	-5.9964	1.000000	.000000	.000000	
.3	1.46343	(-15)	-5.9918	1.000000	.000000	.000000	
.4	.061208	(-12)	-5.9854	1.000000	.000000	.000000	
.5	1.10428	(-12)	-5.9772	1.000000	.000000	.000000	
.6	.0116971	(-9)	-5.9671	1.000000	.000000	.000000	
.7	.085749	(-9)	-5.9552	1.000000	.000000	.000000	
.8	.47992	(-9)	-5.9414	1.000000	.000000	.000000	
.9	2.1847	(-9)	-5.9258	1.000000	.000000	.000000	
1.0	8.4463	(-9)	-5.9081	1.000000	.000000	.000000	
1.1	.028601	(-6)	-5.8886	1.000000	.000000	.000000	
1.2	.086782	(-6)	-5.8671	1.000000	.000000	.000000	
1.3	.24005	(-6)	-5.8436	1.000000	.000000	.000000	
1.4	.61353	(-6)	-5.8181	1.000000	.000000	.000000	
1.5	1.46437	(-6)	-5.7905	1.000000	.000000	.000000	
1.6	3.2921	(-6)	-5.7608	1.000000	.000001	.000000	
1.7	7.0198	(-6)	-5.7289	1.000000	.000001	.000000	
1.8	.0142806	(-3)	-5.6949	1.000000	.000002	.000000	
1.9	.027851	(-3)	-5.6584	1.000000	.000005	.000000	
2.0	.052284	(-3)	-5.6198	1.000000	.000008	.000000	
2.1	.094811	(-3)	-5.5787	1.000000	.000015	.000000	
2.2	.16657	(-3)	-5.5349	1.000000	.000027	.000000	
2.3	.28428	(-3)	-5.4886	1.000000	.000047	.000000	
2.4	.47231	(-3)	-5.4396	1.000000	.000078	.0015	(-6)
2.5	.77039	(-3)	-5.3877	1.000000	.000128	.0041	(-6)
2.6	1.21222	(-3)	-5.3327	1.000000	.000204	.0104	(-6)
2.7	1.7273	(-3)	-5.2746	1.000000	.000319	.0255	(-6)
2.8	2.8530	(-3)	-5.2131	1.000000	.000489	.0597	(-6)
2.9	4.2508	(-3)	-5.1480	1.000000	.000735	.1350	(-6)
3.0	6.2197	(-3)	-5.0792	.999999	.001086	.2947	(-6)

$\ell = 7$

x	s	Δ	cos 2δ	sin 2δ	sin $^2\delta$
.1	.054718 (-24)	-6.9992	1.000000	.000000	.000000
.2	1.7889 (-21)	-6.9969	1.000000	.000000	.000000
.3	.78032 (-18)	-6.9930	1.000000	.000000	.000000
.4	.058079 (-15)	-6.9877	1.000000	.000000	.000000
.5	1.6393 (-15)	-6.9807	1.000000	.000000	.000000
.6	.025043 (-12)	-6.9723	1.000000	.000000	.000000
.7	.25034 (-12)	-6.9622	1.000000	.000000	.000000
.8	1.8339 (-12)	-6.9506	1.000000	.000000	.000000
.9	.0105917(-9)	-6.9374	1.000000	.000000	.000000
1.0	.050692 (-9)	-6.9225	1.000000	.000000	.000000
1.1	.20833 (-9)	-6.9061	1.000000	.000000	.000000
1.2	.75480 (-9)	-6.8881	1.000000	.000000	.000000
1.3	2.4593 (-9)	-6.8684	1.000000	.000000	.000000
1.4	7.3189 (-9)	-6.8471	1.000000	.000000	.000000
1.5	.020140 (-6)	-6.8242	1.000000	.000000	.000000
1.6	.051755 (-6)	-6.7994	1.000000	.000000	.000000
1.7	.125211 (-6)	-6.7729	1.000000	.000000	.000000
1.8	.28710 (-6)	-6.7447	1.000000	.000000	.000000
1.9	.62745 (-6)	-6.7148	1.000000	.000000	.000000
2.0	1.31318 (-6)	-6.6830	1.000000	.000000	.000000
2.1	2.6426 (-6)	-6.6495	1.000000	.000000	.000000
2.2	5.1311 (-6)	-6.6139	1.000000	.000001	.000000
2.3	9.6420 (-6)	-6.5764	1.000000	.000001	.000000
2.4	.017581 (-3)	-6.5370	1.000000	.000002	.000000
2.5	.031377 (-3)	-6.4955	1.000000	.000004	.000000
2.6	.053878 (-3)	-6.4519	1.000000	.000008	.000000
2.7	.083575 (-3)	-6.4061	1.000000	.000013	.000000
2.8	.149956 (-3)	-6.3581	1.000000	.000022	.000000
2.9	.24225 (-3)	-6.3077	1.000000	.000035	.000000
3.0	.38365 (-3)	-6.2549	1.000000	.000056	.000000